Thermal Models in Power System Protection

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ABSTRACT

System protection engineers are familiar with I^2t thermal limitations specified for transformers, motors or generators. However, because of the limitations of electromechanical relaying, thermal protection had to be treated as an overcurrent coordination problem without regard to dynamics or thermal history involved.

Today time-discrete forms of differential equations are easily implemented in microprocessor-based relays. Consequently, algorithms based on real-time computing power add a new dimension for accurate thermal protection using thermal models. This paper is a tutorial on the thermal models used in power system protection, their parameters, and how they are derived from available application data.

INTRODUCTION

This paper reviews the nature of the thermal models used to compute and monitor the temperature rise for the thermal protection of motors, transformers, overhead lines, and cables. The basic model is developed from the equation for temperature rise caused by watts loss on a conductor. The paper then establishes the relation of the model to the I^2t characteristic as a representation of a limiting temperature.

The paper then reviews the more complex models of hottest-spot temperature rise for oil-cooled transformers as specified by IEEE STD C57.91-1995, and overhead lines as given in IEEE Std 738 -1993. Next the paper discusses the calculation of thermal resistance and thermal capacitance required for the thermal models of buried cables. Finally, it lists conclusions drawn on the possibilities and outlook for thermal protection for machines, transformers, overhead lines, and cables.

OVERCURRENT RELAYS VS THE THERMAL MODEL

By definition[1], an overcurrent relay produces an inverse time-current characteristic by integrating a function of current F(I) with respect to time. The F(I) is positive above and negative below a predetermined input current called the pickup current. Pickup current is the current at which integration starts positively, and the relay produces an output when the integral reaches a predetermined set value. The F(I) represents the velocity of an induction disk and the integral represents its travel. Figure 1 shows the overcurrent relay represented as an analog circuit. The disk travel is represented by a voltage V. The function is $F(I) = I^2 - 1$ which produces an extremely-inverse characteristic where I is the input in per unit of the pickup current.

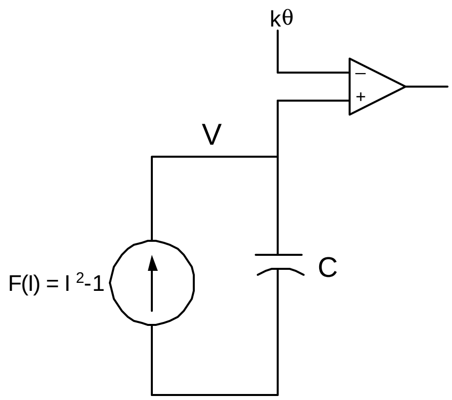


Figure 1: Analog of an Overcurrent Relay

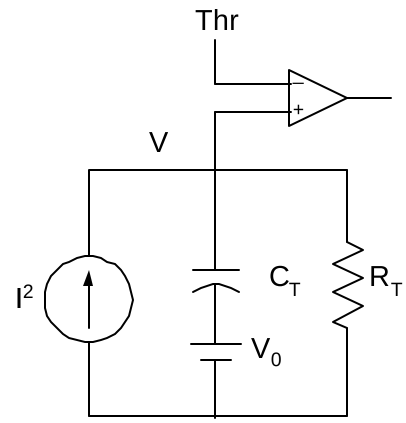


Figure 2: Analog of a Thermal Model

Figure 2 shows a thermal model represented by an analog circuit. The voltage V in the thermal model represents the temperature rise in I^2t above an initial temperature V_0. In Figure 2, V has a steady state value for every value of input current and a trip is asserted only if V exceeds the trip threshold Thr. The threshold is set at the limiting temperature of the protected element. By contrast, in Figure 1, the voltage V has no steady state and rises to exceed the trip threshold kθ for any I > 1 and resets to zero for any current I < 1. The overcurrent trip time is set by means of a time dial that increases the travel by modifying factor k. The thermal model trip time is determined by the time constant R_TC_T, which is set to equal the thermal time-constant of the protected element.

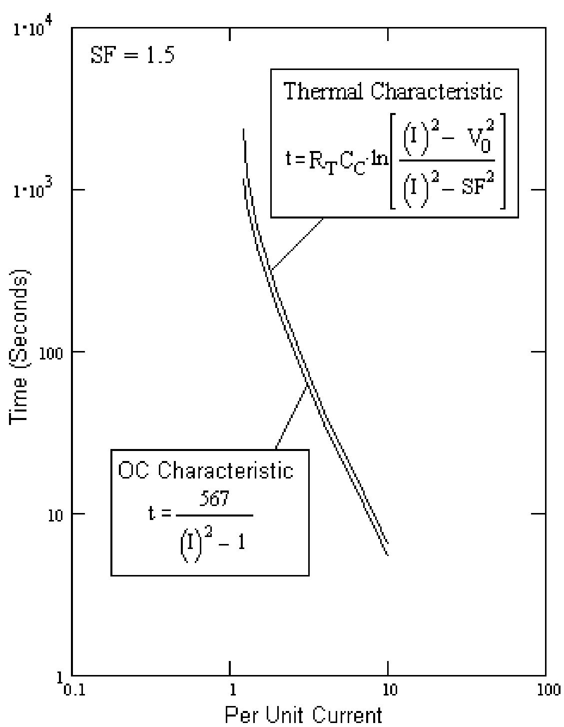


Figure 3: Overcurrent Coordination With a Thermal Characteristic

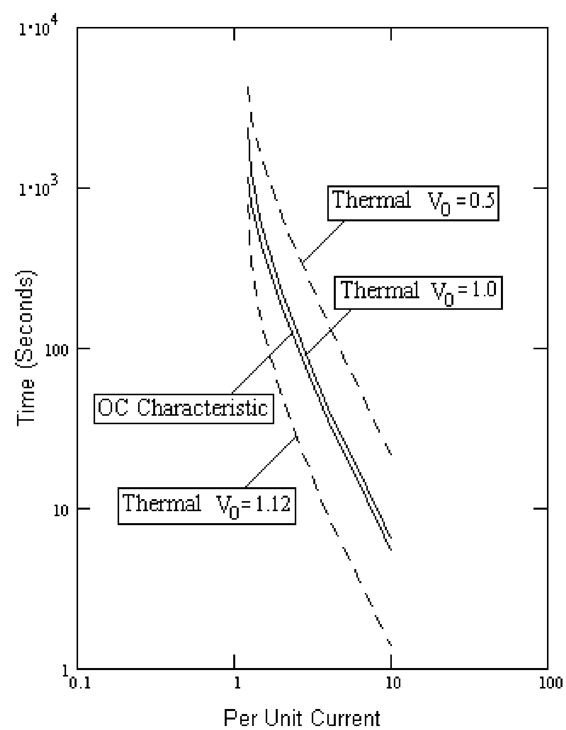


Figure 4: Thermal Characteristic Variation With Initial Condition V_0

Figure 3 shows that an overcurrent characteristic can emulate the shape of the thermal characteristic and can be closely coordinated with it. However, Figure 4 shows that the coordination is valid only for the initial temperature V_0 equal to 1 per unit. When a temporary overload raises the temperature, V_0 increases and the time to reach the damaging temperature decreases as shown in Figure 4. Consequently, during a recurring overload, the damaging temperature occurs before the overcurrent relay can operate. In addition, the coordination prevents the use of the added thermal capacity available when temperature V_0 is less than 1.0.

Consequently, only the thermal model can account for thermal history and accurately track the excursions of conductor temperatures.

DETERMINING A MOTOR THERMAL MODEL

The motor thermal model accounts for the slip dependent I^2r heating of both positive- and negative-sequence current and is defined by motor nameplate and thermal limit data. This mathematical model calculates the motor temperature in real time. The temperature is then compared to thermal limit trip thresholds to prevent overheating from overload, locked rotor, too frequent or prolonged starts, or unbalanced current.

What data defines the thermal model? Full load speed, the locked rotor current and torque, and the thermal limit time define the model.

What does torque have to do with the thermal model?

The I^2r heat source and two trip thresholds are identified by the motor torque, current, and rotor resistance versus slip shown in Figure 5. Figure 5 shows the distinctive characteristic of the induction motor to draw excessively high current until the peak torque develops near full speed. Also, the skin effect of the slip frequency causes a high locked rotor resistance, R_1, which decreases to a low running value, R_0, at rated slip.

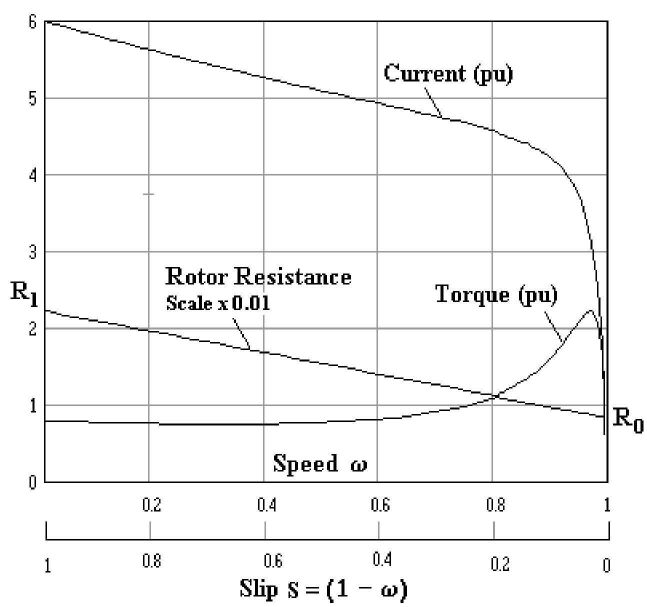


Figure 5: Current, Torque, and Rotor Resistance of an Induction Motor Versus Speed

A typical starting current of six times the rated current and a locked rotor resistance R_1 of three times the value of R_0 causes the I^2r heating to be 6^2 x 3 or 108 times normal. Consequently, an extreme temperature must be tolerated for a limited time to start the motor. A high emergency I^2t threshold is specified by the locked rotor limit during a start, and a second lower threshold for the normal running condition is specified by the service factor. Therefore, the thermal model requires a trip threshold when starting, indicated by the locked rotor thermal limit, and a trip threshold when running, indicated by the service factor.

How is the heating effect of the positive- and negative-sequence current determined? The positive-sequence rotor resistance is plotted in Figure 5 and is calculated using current I, torque Q_M, and slip S in the following equation:

$$R_r = \frac{Q_M}{I^2} S \tag{1}$$

It is represented by the linear function of slip shown in Figure 5. The positive-sequence resistance R_{r+} is a function of the slip S:

$$R_{r+} = (R_1 - R_0) S + R_0 \tag{2}$$

The negative-sequence resistance R_{r-} is obtained when S is replaced with the negative-sequence slip (2-S):

$$R_{r-} = (R_1 - R_0)(2 - S) + R_0 \tag{3}$$

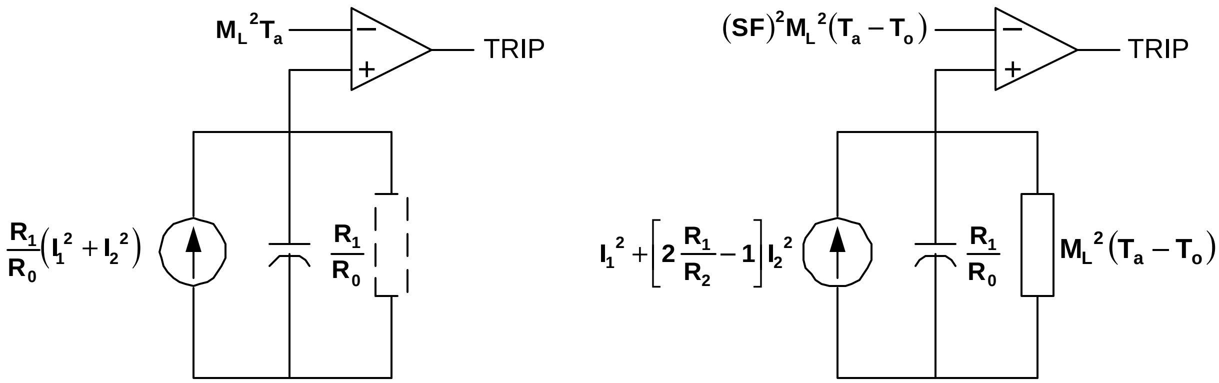
We obtain factors expressing the heating effect of positive- and negative-sequence current by dividing Equations 2 and 3 by the running resistance R_0. Consequently, for the locked rotor case, and where R_1 is typically three times R_0, the heating effect for both positive- and negative-sequence current is three times that caused by the normal running current.

$$\frac{R_{r+}}{R_0}\Big|_{S=1} = \frac{R_{r-}}{R_0}\Big|_{S=1} = \frac{R_1}{R_0} = 3 \tag{4}$$

For the running case, the positive-sequence heating factor returns to one, and the negative-sequence heating factor increases to 5:

$$\frac{R_{r+}}{R_0}\Big|_{S=0} = 1 \quad \frac{R_{r-}}{R_0}\Big|_{S=0} = 2\left(\frac{R_1}{R_0}\right) - 1 = 5 \tag{5}$$

These factors are the coefficients of the positive and negative currents of the heat source in the thermal model shown in Figure 6.



(a) Starting State I >2.5 pu (b) Running State I <2.5 pu

Figure 6: States of the Thermal Model

Starting and Running States of the Motor Thermal Model

Because of its torque characteristic, the motor must operate in either a high current starting state or be driven to a low current running state by the peak torque occurring at about 2.5 per unit current. The thermal model protects the motor in either state by using the trip threshold and heating factors indicated by the current magnitude. The two states of the thermal model are shown in Figure 6. The thermal model is actually a difference equation executed by the microprocessor. However, it can be represented by the electrical analog circuit shown in Figure 6. In this analogy, the heat source is represented by a current generator, the temperature is represented by voltage, and thermal resistance and capacitance are represented by electrical resistance and capacitance. The parameters of the thermal model are defined as follows:

R_1 = Locked rotor electrical resistance (per unit ohms)
R_0 = Running rotor electrical resistance also rated slip (per unit ohms)
I_L = Locked rotor current in per unit of full load current
T_a = Locked rotor time with motor initially at ambient
T_o = Locked rotor time with motor initially at operating temperature

TRANSFORMER THERMAL MODEL

The transformer thermal model satisfies the requirements specified in ANSI standard C57.92-1995 [3] to provide transformer overload protection. The model consists of two exponential equations and non-linear time-constants determined from transformer data.

The thermal model calculates the following temperatures shown in Figure 7 for reference:

θ_o Top-oil rise over ambient temperature, °C

θ_g Hottest-spot conductor rise over top-oil temperature, °C

θ_{hs} Hottest-spot winding temperature, °C

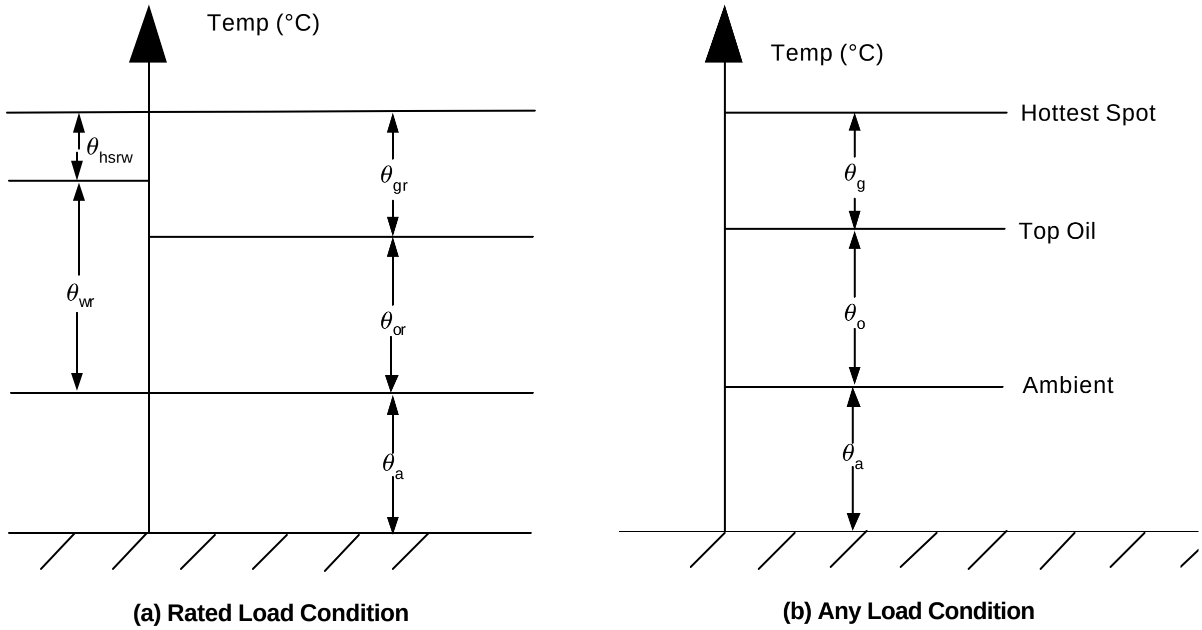


Figure 7: Transformer Temperatures for Different Load Conditions

Top-Oil Rise Over Ambient Temperature, θ_o

When a constant load is applied throughout the time interval Δt, the model calculates the top-oil rise over ambient temperature at the end of the interval, according to the following expression:

$$\theta_o = (\theta_{ou} - \theta_{oi}) \cdot \left(1 - e^{\frac{-\Delta t}{60 \cdot T_o}}\right) + \theta_{oi} \tag{6}$$

where:

- θ_{ou} Ultimate top-oil rise over ambient temperature for any load, °C
- θ_{oi} Initial top-oil rise over ambient temperature at the start time of the interval, °C
- T_o Oil time constant of transformer, in hours

The increment Δt defines the time interval between calculations. The recommended interval is 10 minutes.

Ultimate Top-Oil Rise Over Ambient Temperature

The model calculates the ultimate top-oil rise over ambient temperature due to constant loads according to the following expression:

$$\theta_{ou} = \left[\frac{(K^2 \cdot R + 1)}{(R + 1)}\right]^n \cdot \theta_{or} \tag{7}$$

where:

K Load expressed in per unit of transformer nameplate rating according to the cooling system in service (maximum phase current divided by the nominal current)

R Ratio of load loss at rated load to no-load loss

n An empirically derived oil exponent

θ_{or} Top-oil rise over ambient temperature at rated load, °C

Oil Time Constant of the Transformer

The following expression determines the thermal top-oil time constant of the transformer for any n (oil exponent accounting for viscosity change with temperature) value and for any load value:

$$T_o = T_r \cdot \left[\frac{\frac{\theta_{ou}}{\theta_{or}} - \frac{\theta_{oi}}{\theta_{or}}}{\left(\frac{\theta_{ou}}{\theta_{or}}\right)^{\frac{1}{n}} - \left(\frac{\theta_{oi}}{\theta_{or}}\right)^{\frac{1}{n}}} \right] \tag{8}$$

where:

T_r Thermal time constant at rated load with initial top-oil temperature equal to ambient temperature

If the T_r time constant is not known, the following expression determines the time constant:

$$T_r = C \cdot \left(\frac{\theta_{or}}{P_r} \right) \tag{9}$$

where:

C Transformer thermal capacity (watt-hours/degree)
= 0.06 · (weight of core and coil assembly in pounds)
+ 0.04 · (weight of tank and fitting in pounds)
+ 1.33 · (gallons of oil)

or

C Transformer thermal capacity (watt-hours/degree)
= 0.0272 · (weight of core and coil assembly in kilograms)
+ 0.01814 · (weight of tank and fitting in kilograms)
+ 5.034 · (liters of oil)

P_r Total loss at rated load (watts)

The model adds the ambient temperature, θ_a, to the top-oil rise over ambient temperature, θ_o, to obtain the top-oil temperature, θ_{TO}. The top-oil temperature at the end of the time interval, Δt, is:

$$\theta_{TO} = \theta_o + \theta_a \tag{10}$$

Hottest-Spot Conductor Rise Over Top-Oil Temperature, qg

When a constant load is applied throughout time interval Δt, the model calculates the hottest-spot conductor rise over top-oil temperature at the end of the interval, according to the following expression:

$$\theta_g = \left(\theta_{gu} - \theta_{gi}\right) \cdot \left(1 - e^{\frac{-\Delta t}{60 \cdot T_{hs}}}\right) + \theta_{gi} \tag{11}$$

where:

- θ_{gu} Ultimate hottest-spot rise over top-oil temperature for any load K, °C
- θ_{gi} Initial hottest-spot rise over top-oil temperature at the start time of the interval, °C
- T_{hs} Winding time constant of hottest-spot, in hours

Ultimate Hottest-Spot Rise Over Top-Oil Temperature

The model calculates the ultimate hottest-spot rise over top-oil temperature due to constant loads according to the following expression:

$$\theta_{gu} = K^{2m} \cdot \theta_{gr} \tag{12}$$

where:

- m An empirically derived winding exponent
- θ_{gr} Hottest-spot conductor rise over top-oil at rated load, °C

Hottest-Spot Conductor Rise Over Top-Oil at Rated Load

When the transformer manufacturer does not provide the hottest-spot conductor rise over top-oil at rated load, you can calculate it as follows:

$$\theta_{gr} = \theta_{wr} + \theta_{hsrw} - \theta_{or} \tag{13}$$

where:

- θ_{wr} Average winding temperature rise over ambient temperature at rated load, °C
- θ_{hsrw} Hottest-spot conductor rise over average winding rise, °C

Hottest-Spot Winding Temperature, θ_{hs}

When a constant load is applied throughout time interval Δt, the relay calculates the hottest-spot winding temperature at the end of the interval, according to the following expression:

$$\theta_{hs} = \theta_a + \theta_o + \theta_g \tag{14}$$

Thus it takes two exponential expressions, thermal capacitance and times constants, calculated using transformer nameplate and test data, to determine the hottest-spot temperature under a specified load. The model is given in clause 7 of the standard and is based on top-oil temperature as a starting point. An alternative, more accurate, model is given in Annex G of the ANSI standard using bottom-oil temperature as a starting point. This model, although more accurate for large transient loads, presents a difficulty in that only top-oil temperature is available from most manufacturers. Annex G introduces duct oil temperature which may be higher than top-oil temperature, giving a true hottest-spot located in the duct.

Table 1: Exponents Used in Temperature Equations

Type of Cooling	m	n
OA	0.8	0.8
FA	0.8	0.9
Non-directed FOA or FOW	0.8	0.9
Directed FOA or FOW	1.0	1.0

HEATING OF OVERHEAD LINES

The heat balance equation for over-head conductors takes into account heat gain from the watt loss, convection loss due to wind velocity, heat loss due to radiation, and heat gain from solar radiation. These parameters are defined in subsequent equations with tables and polynomial equations provided to determine constants as explained in IEEE Std 738-1193 [4]:

$$q_c + q_r + mC_p \frac{dT_c}{dt} = q_s + I^2 \cdot R(T_c) \tag{15}$$

where:

- q_c Convected heat loss
- q_r Radiated heat loss
- q_s Solar gain
- mC_p Total conductor heat capacity
- $R(T_c)$ = Conductor electrical resistance

Forced Convection Heat

Equation (16) gives the heat loss in a conductor for low speed wind velocity. However, Equation (16) underestimates losses from high wind speeds. Equation (17) is needed for high speed wind velocity:

$$q_{c1} = \left[1.01 + 0.371\left(\frac{D\rho_f V_w}{\mu_f}\right)^{0.52}\right] \cdot k_f \cdot (T_c - T_a) \tag{16}$$

$$q_{c2} = 0.1695\left(\frac{D\rho_f V_w}{\mu_f}\right)^{0.6} \cdot k_f \cdot (T_c - T_a) \tag{17}$$

where:

D = Conductor diameter (in)

ρ_f = Density of air (lb/ft^3)

μ_f = Absolute viscosity of air (lb/ft h)

V_w = Velocity of the air stream (ft/h)

T_c = Conductor temperature (°C)

T_a = Ambient temperature (°C)

k_f = Thermal conductivity of air, W/ft (°C)

The physics governing cooling caused by wind velocity, unfamiliar to many protection engineers, are given in tables or polynomials provided in IEEE Std 738-1193 [4].

Radiated Heat Loss

Radiated loss depends on the emissivity, ε, (0.23 to 0.91) which is a property of the surface and diameter of the conductor and is proportional to the 4th power of the absolute temperature:

$$q_r = 0.138 \cdot D \cdot \varepsilon \cdot \left[\left(\frac{T_c + 273^o}{100}\right)^4 - \left(\frac{T_a + 273^o}{100}\right)^4\right] \tag{18}$$

Solar Gain

Heat gain in the conductor from the sun is given by Equation (19) with the angle defined as in Equation (20).

$$q_S = \alpha \cdot Q_S \cdot \sin(\theta)A \tag{19}$$

$$\theta = \cos^{-1}\left[\cos(H_c)\cos(Z_c - Z_l)\right] \tag{20}$$

where:

θ = Effective angle of incidence of the sun's rays (degrees)

α = Solar absorptivity (0.23 to 0.91)

Q_s = Total solar and sky radiated heat flux, (W/ft^2)

A = Projected area of the conductor, (ft^2/ft)

H_c = Altitude of sun (degrees)

Z_c = Azimuth of the sun (degrees)

Z_l = Azimuth in line degrees

Tables and polynomial equations for determining these constants are provided.

Conductor Electrical Resistance

The values conductor resistance at high temperature $T_{high,}$ and low temperature T_{low} may be taken from tabulated values in reference [5]. The resistance can then be determined by linear interpolation:

$$R(T_c) = \left[\frac{R(T_{high}) - R(T_{low})}{T_{high} - T_{low}}\right] \cdot (T_{high} - T_{low}) + R(T_{low}) \tag{21}$$

The heat loss equations reviewed above have been successfully implemented as the thermal element in a microprocessor-based line protection relay.

POWER CABLE THERMAL MODEL

Figure 8, the thermal model of a cable, shows the thermal resistance and the distribution of the thermal capacitance for all layers of the cable. The model of a multi-layered cable is characterized by its complexity and the relative difficulty of determining its parameters. Although the thermal capacitance and resistance are available in standards and specifications of transformers [3], motors [2], and overhead lines [4], they must be calculated for the cable using physical parameters [5].

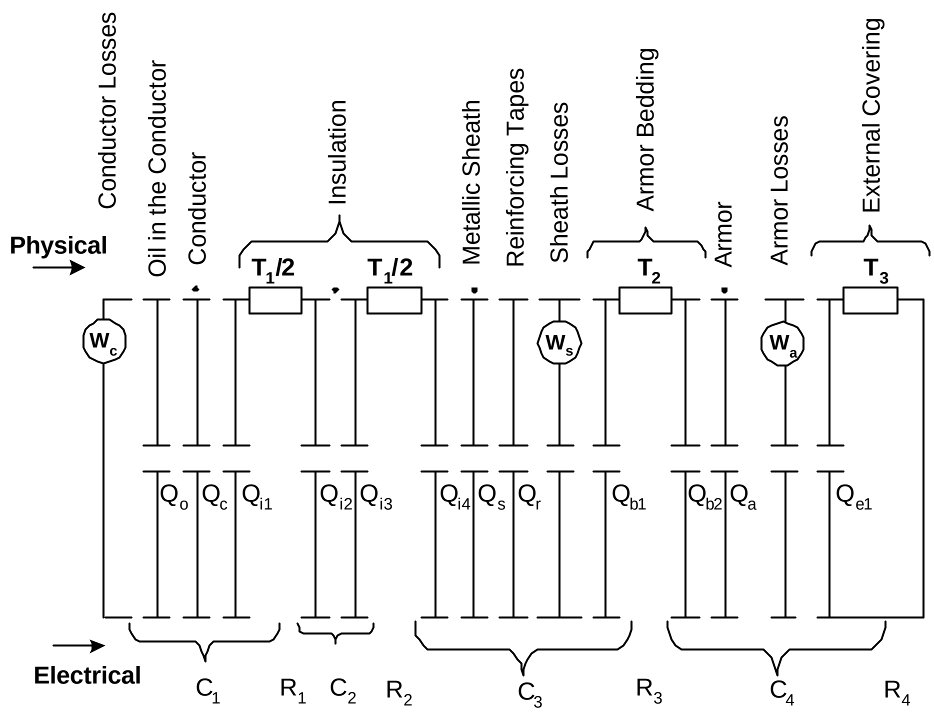


Figure 8: Thermal Model of a Cable

Thermal Resistance of a Cylindrical Layer

The thermal resistance of the insulation layers of the cable must be calculated. The equation here is for a cylindrical layer of insulation. Thermal resistance must be calculated for each layer of insulation, and armor, and for the screen layers, sheath and/or oil layers that make up the cable. T is the IEC symbol used for thermal resistance:

$$T = \frac{\rho_{th}}{2\pi} \ln \frac{r_2}{r_1} \tag{22}$$

where:

ρ_{th} Thermal resistivity of the material K m/W

r1 Inside radius (m)

r2 Outside radius (m)

Thermal Capacitance

The thermal capacitance must be calculated for each layer of insulation, armor, sheath, and/or oil that makes up the cable outer layers. The thermal capacitance is obtained by multiplying the volume of the material by its specific heat. Q is the IEC symbol used for thermal capacitance.

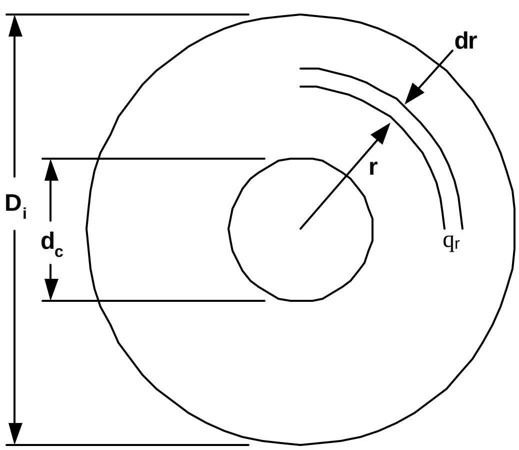
$$Q_{th} = V \cdot c = \frac{\pi}{4}\left(D_2^2 - D_1^2\right) \tag{23}$$

where:

V	Volume (m^3)
c	Specific heat ($J/(m^3 \cdot K)$)
D_1	Inside diameter (m)
D_2	Outside diameter (m)

As Anders describes [6], "The thermal capacity of the insulation is not a linear function of the thickness of the dielectric. To improve the accuracy of the approximate solution using lumped constants, Van Wormer, in 1955, proposed a simple method of allocating the thermal capacitance between the conductor and the sheath so that the total heat stored in the insulation is represented." The derivation, as shown in Figure 9, is based on the assumption that the temperature distribution in the insulation follows the steady state logarithmic distribution during the transient. A three-node representation of the dielectric of a cable with thermal capacitance distributed using Van Wormer coefficients is shown in Figure 10.

The thermal model in Figure 8 may suggest that measured current can be used to calculate temperature in a cable. However, the density and resistivity of the soil back-fill is crucial in determining the temperature rise. The backfill temperature must be monitored.



$$p = \frac{1}{2\ln\left(\frac{D_i}{d_c}\right)} - \frac{1}{\left(\frac{D_i}{d_c}\right)^2 - 1}$$

Figure 9: Logarithmic Distribution of Thermal Capacitance in Annular Insulation

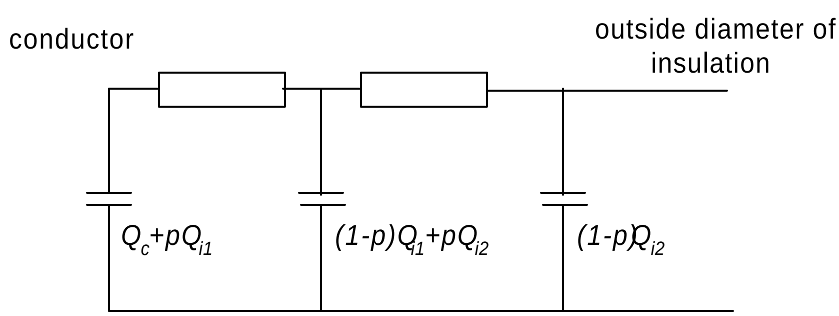


Figure 10: Three-Node Representation Using Van Wormer Coefficient

CONCLUSIONS

1. A long-time extremely-inverse overcurrent relay can emulate the shape of a thermal characteristic and can be coordinated to give thermal protection for a fixed initial condition. However, once heating occurs the overcurrent relay cannot prevent thermal damage for cyclic overloads.

2. A thermal model used for motor protection is defined by available nameplate, torque and thermal limits specified for the motor. The model accurately accounts for the thermal history and accurately tracks temperature throughout the operating cycle.

3. An IEEE standard [3] defines thermal models for calculating transformer hottest-spot winding temperature, using top-oil temperature as the starting point and involving non-linear time constants. The standard gives an alternate method using bottom-oil temperature that is more accurate but requires test data that is rarely available by specification.

4. A comprehensive IEEE standard gives the thermal model for overhead transmission lines. The heat balance equation includes convected and radiated heat loss and solar gain, as well as total conductor thermal capacity and watt loss. The standard provides polynomials for tabulated constants to aid computation. This thermal model has been integrated as an element in a line protection relay.

5. The thermal model for a multilayer cable is characterized by complexity and by the difficulty of determining the thermal parameter. Although thermal capacitance and resistance values are available for transformers, motors, and overhead lines, for cables they must be calculated from physical parameters.

REFERENCES

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[2] IEEE Std 620 - 1996, IEEE Guide for the Presentation of Thermal Limit Curves for Squirrel Cage Induction Machines.

[3] IEEE Std C57.91 -1995, IEEE Guide for Loading Mineral-Oil-Immersed Transformers.

[4] IEEE Std 738-1993, IEEE Standard for Calculating the Current-Temperature Relationship of Bare Overhead Conductors.

[5] *Aluminum Electrical Conductor Handbook* 2nd ed. Washington DC: The Aluminum Association.

[6] George J. Anders, Ratings of Electrical Power Cables - Ampacity Computations for Transmission, Distribution and Industrial Applications, IEEE Press 1997.

ANNEX A : THE HEAT EQUATION

The thermal model and its relation to an I^2t time-current characteristic is derived from the differential equation for temperature rise caused by current in a conductor:

$$I^2 r = C_T \frac{d\theta}{dt} \tag{A1}$$

where:

I^2r Is the input in watts (conductor loss)

C_T Is the thermal capacity of the conductor in watt-sec./°C

$d\theta/dt$ Is the rate of change of temperature in °C/second

Integrate the equation to find the temperature:

$$\theta = \frac{1}{C_T} \int_0^t I^2 r \cdot dt \tag{A2}$$

Therefore the temperature rise for constant input watts is:

$$\theta = \frac{I^2 r}{C_T} t \tag{A3}$$

where θ is the temperature in °C. The heat equation can be represented by an analog circuit consisting of a current generator feeding a capacitor. The current is numerically equal to the watts; the capacitor equals the thermal capacitance. The voltage charge accumulated by the capacitor represents the temperature rise over ambient caused by the watts. The temperature can be expressed in per unit and be plotted versus per unit current as a time-current characteristic. To do this let:

$$I = m \cdot I_{rated} \tag{A4}$$

and substitute for I in Equation (A4):

$$\theta = \frac{(m \cdot I_{rated})^2 r}{C_T} t \tag{A5}$$

Divide Equation (A5) through by: $\frac{(I_{rated})^2 r}{C_T}$ to obtain:

$$\theta \frac{C_T}{(I_{rated})^2 r} = m^2 t \tag{A6}$$

By expressing the left side of (A6) as Θ the equation can be written simply as:

$$\Theta = m^2 t \tag{A7}$$

The above equations show that an I^2t curve can represent a thermal limit. The curve represents a specific temperature expressed in seconds and is a straight line which has a slope of 2 when plotted on log-log paper. If at locked rotor current M_L the thermal limit time is T_A , then from (A7) the temperature represents the rise over ambient:

$$\Theta_L = M_L^2 T_A \tag{A8}$$

If at the locked rotor current M_L the time to reach the thermal limit is T_O with the conductor initially at operating temperature, then Equation (A8) becomes:

$$M_L^2 T_A = M_L^2 T_O + \Theta_O \tag{A9}$$

and the operating temperature in terms to M_L, T_A, and T_O is:

$$\Theta_O = M_L^2 (T_A - T_O) \tag{A10}$$

Equation (A7) is the familiar I^2t characteristic that plots as a straight line of slope on log-log paper. The plot shown in Figure A1 represents a specific limiting temperature. The operating temperature, represented by Equation (A10) , is caused by one per unit current flowing in the thermal resistance of the conductor. Consequently, Equation (A10) is the thermal resistance as shown in the electrical analog shown in Figure A2.

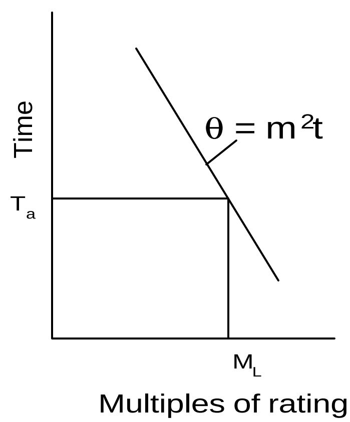

Figure A1: Thermal Limit Curve

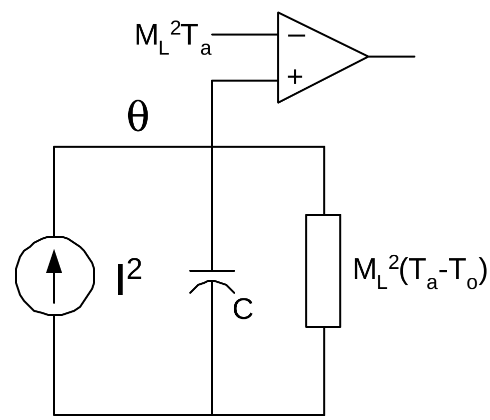

Figure A2: Thermal Model