Decision Analysis Applied to Protective Relaying

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Abstract—**Protection engineers often must select a relaying scheme or system while answering multiple competing objectives. The choice of any system will have several alternatives. The best solution is often not clear because no single alternative will satisfy all objectives. Decision analysis methods use rigorous techniques to determine the best alternative that solves a problem with multiple competing objectives. The decision analysis process this paper describes assigns a weight to the importance of each objective. The paper then describes a technique for ranking each of the alternatives in terms of the best consideration of all desired objectives.**

This paper presents a customized methodology for relay system selection. For illustration, a line protection system example shows use of the methodology in evaluating various alternatives, including two identical schemes, two similar schemes with different algorithms, and two completely dissimilar schemes with different operating principles.

Different objectives this paper evaluates include the following:

- **Fast fault clearing times**
- **High dependability**
- **High security**
- **Best economy**
- **Ease of setting and applying**
- **Preferred relay communications channel**

The paper includes methods for incorporating and weighting additional objectives.

This paper describes a decision analysis method that uses a multiattribute utility theory and also discusses uncertainties in meeting the objectives. While recognizing that engineering a protective relay system is an art, this paper provides a basis in science for system selection.

I. Introduction

Protection engineers frequently make decisions that involve tradeoffs among alternatives. Should a protection scheme lean towards greater dependability or greater security? Are fast clearing times necessary, or will a less costly protection scheme be acceptable? If a communications-aided protection scheme is necessary, what type of communications must we apply? It is possible to rate alternatives as to how they satisfy certain attributes we determine to be important. The protection engineer cannot apply a single protective relaying scheme that is best for each attribute, so tradeoffs are necessary. The number of alternatives available, the complexity of the attributes that each alternative is to be rated against, and any uncertainties in meeting these attributes all contribute to the degree of difficulty in making a decision.

Decision analysis techniques in engineering and business have been in use for decades. The electric power industry has also applied these techniques. Decisions regarding the addition of new generation to an electric system are an example of where decision analysis techniques are frequently used.

Strategic decisions require taking a structured approach with a formal decision-making process. One such multiobjective decision analysis technique applies multiattribute utility theory. Kirkwood [1] lists the following five steps in the decision-making process:

1. Specify objectives and scales for measuring achievement with respect to these objectives.
2. Develop alternatives that potentially might achieve each objective.
3. Determine how well each alternative achieves each objective.
4. Consider tradeoffs among the objectives.
5. Select the alternative that, on balance, best achieves the objectives, taking into account uncertainties.

Hammond, Keeney, and Raiffa [2] use the PrOACT acronym to structure the decision making process as follows:

Problem
Objectives
Alternatives
Consequences
Tradeoffs

The key to making a good decision is to first identify what the problem is that must be solved. Once we define the problem, we can specify objectives. We then determine potential alternatives that attempt to meet these objectives. Selecting each alternative results in consequences in terms of meeting the stated objectives. Tradeoffs will be necessary if no single alternative is best at meeting all objectives. The authors also consider uncertainty, risk tolerance, and linked decisions.

II. Decision Analysis Without Uncertainty

The first step in solving any decision problem is to define the problem. A well-defined problem statement determines the alternatives we will consider and how we will evaluate these alternatives.

The next step is to specify the objectives we want to meet with the solution to our problem. The objectives are important because they form the basis for evaluating the alternatives we are considering. It is critical to spend a significant amount of time considering the objectives. Objectives should be ends in themselves and not a means to an end. Keep asking “Why?” until you can no longer answer this question.

Once you have specified the objectives, you must determine a means of measuring achievement with respect to each objective. Kirkwood [1] defines an evaluation measure as "a measuring scale for the degree of attainment of an objective." Define a single dimensional value function for each evaluation measure. Use a piecewise linear value function when the evaluation measure has a small number of possible values. Use an exponential value function when the evaluation measure has an infinite number of possible values. Apply value functions when there is no uncertainty in how each alternative meets the stated objectives. Each value function ranges from zero to one. Section III discusses a process for determining the appropriate value function. Section V considers decisions with uncertainty.

The next step is determination of a number of possible alternatives that attempt to solve the problem by meeting the stated objectives. While asking "Why?" is helpful in defining the objectives, asking "How?" creates alternatives from the objectives. Hammond, Keeney, and Raiffa [2] list several other suggestions for determining good alternatives:

- Use your objectives—ask "How?"
- Challenge constraints
- Set high aspirations
- Do your own thinking first
- Learn from experience
- Ask others for suggestions
- Give your subconscious time to operate
- Create alternatives first; evaluate them later
- Never stop looking for alternatives

We must then determine the relative importance of each of the objectives (attributes). For each objective we have defined single dimensional value functions. We now assign weights to each single dimensional value function. The following equation defines an overall value function.

$$v(X_1, X_2, ..., X_n) = w_1 v_1(X_1) + w_2 v_2(X_2) + ... + w_n v_n(X_n) \quad (1)$$

where $v_n(X_n)$ is the single dimensional value function for attribute X_n, and w_n is the weight for this attribute. We chose the weights to satisfy the following equation.

$$w_1 + w_2 + ... + w_n = 1 \quad (2)$$

Section IV discusses the selection of the weights.

In the final step, we use (1) to calculate an overall score for each of the alternatives. The score ranges from zero, when each of the attributes is at its least preferred level, to one, when each of the attributes is at its most preferred level. The alternative with the greatest overall score is the preferred choice. Sensitivity analysis we perform by varying the weights provides a means of determining how the relative importance of each attribute affects the decision.

III. Determining Value Functions

Use a piecewise linear function when the evaluation measure for an attribute contains a few number of possible scores. The piecewise linear function should vary from zero to one. We assign a value within this range to each of the evaluation measure scores and account for relative improvements between each evaluation measure score. For example, going from the least preferred score to the next highest score can result in twice the increase in value as in going from the second highest score to the highest score. Fig. 1 shows an example of a piecewise linear value function.

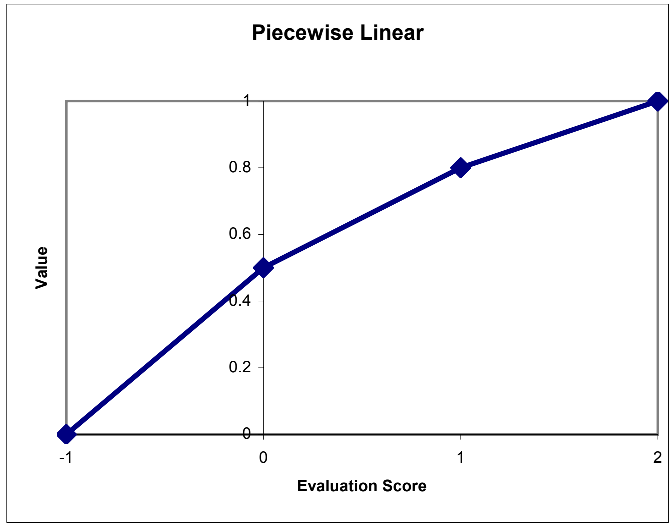

Fig. 1. Example of a piecewise linear value function

Use an exponential value function when the number of possible scores is infinite. There are two types of exponential functions: monotonically increasing over the evaluation score and monotonically decreasing over the evaluation score. The value function will be as follows for cases where one prefers a higher evaluation measure.

$$v(x) = \frac{1 - \exp[-(x - Low)/\rho]}{1 - \exp[-(High - Low)/\rho]}, \quad \rho \neq \infty$$
$$v(x) = \frac{x - Low}{High - Low}, \quad \rho = \infty \quad (3)$$

where ρ is the exponential constant that will determine the shape of the exponential curve.

The value function will be as follows for cases where one prefers a lower evaluation measure.

$$v(x) = \frac{1 - \exp[-(High - x)/\rho]}{1 - \exp[-(High - Low)/\rho]}, \quad \rho \neq \infty$$
$$v(x) = \frac{High - x}{High - Low}, \quad \rho = \infty \quad (4)$$

Apply (3) when you prefer a higher evaluation measure, such as when dollars saved is the attribute. Apply (4) when you prefer a lower evaluation measure, such as when cost in dollars is the attribute. Fig. 2 and Fig. 3 show examples of these exponential value functions.

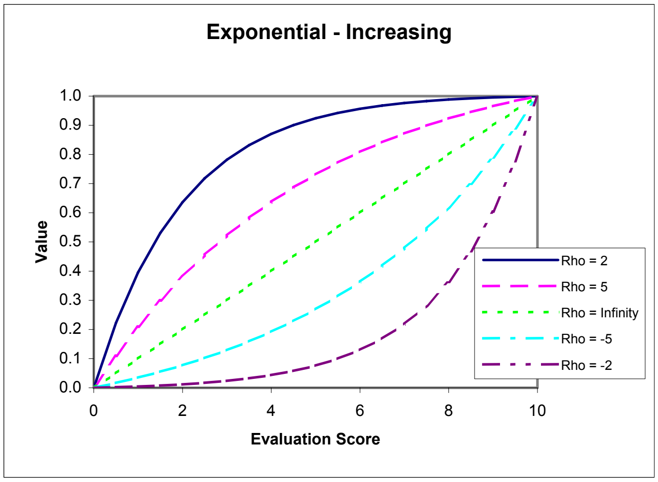

Fig. 2. Monotonically increasing exponential value function

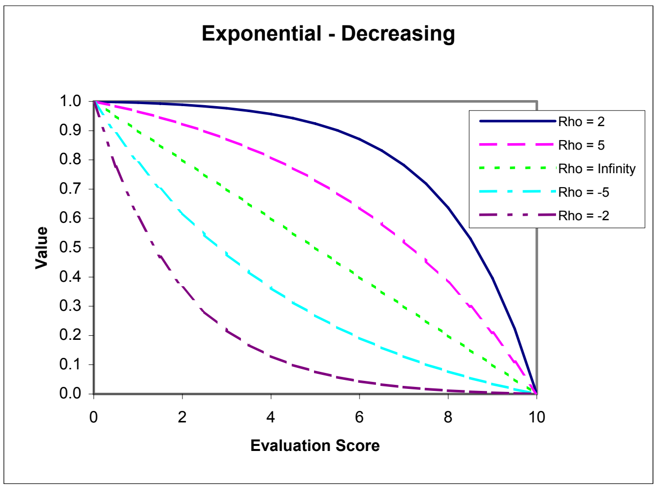

Fig. 3. Monotonically decreasing exponential value function

The exponential constant ρ will range in value typically from one-tenth of the range of possible evaluation scores to ten times the range of possible evaluation scores. Consider what we call the midvalue of the range of evaluation scores to determine the exponential constant ρ for a particular single dimensional value function. We define the midvalue of the range to be the evaluation score with a value of 0.5. When we know the midvalue and two endpoints in the range of evaluation scores, we can solve (3) or (4) numerically to determine the exponential constant ρ. Kirkwood [1] presents a procedure for determining this constant. This procedure is in the appendix.

IV. Determining Weights

The weights we use in the overall value function (1) determine the relative importance of each attribute we consider in the decision. Each single dimensional value function can take a value of between zero and one. We can use this property, along with (2), to determine the appropriate weight for each attribute. The weight for each attribute equals the increment in value we receive by moving the evaluation score on that attribute from its least preferred level to its most preferred level.

A procedure for determining the weights is as follows [1]:

1. Swing the evaluation score on each attribute from its least preferred score to its most preferred score, and place these increments in order of successively increasing value increments.
2. Quantitatively scale each of these value increments in multiples of the smallest value increment.
3. Set the smallest value increment so that the sum of all value increments is one.
4. Use the results of Step 3 to determine the weights for all of the attributes.

V. Decision Analysis With Uncertainty

Use the probability concept of expected value in cases where the evaluation measure for one or more of the attributes can have uncertainty. For attributes that can take on discrete values, we define the expected value as follows:

$$EV_1 = P_1 v_1(x_1) + P_2 v_1(x_2) + \ldots + P_n v_1(x_n) \tag{5}$$

where P_n is the probability that the single dimensional value function for Attribute #1 will have value $v_1(x_n)$.

Use the cumulative probability distribution function to calculate expected values for continuous uncertain quantities. Fig. 4 shows an example of a cumulative probability distribution function.

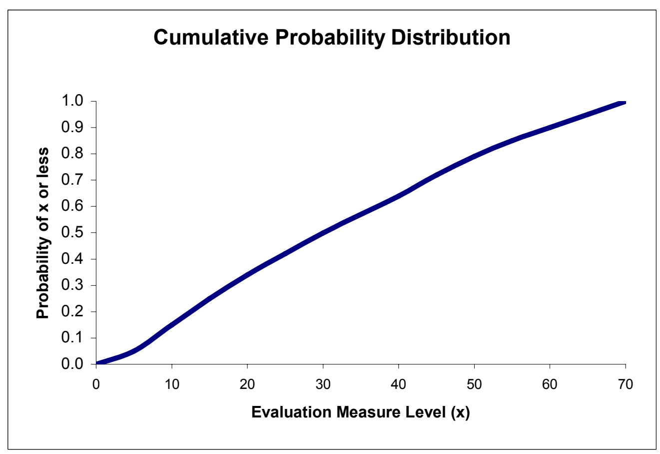

Fig. 4. Cumulative probability distribution function

An exact calculation involves differentiation of this function and then integration of the evaluation measure "x" times this derivative. However, we can use two different methods to approximate the expected value. The extended Pearson-Tukey approximation assigns probabilities of 0.185 at both the 0.05 and 0.95 fractile levels of the cumulative probability distribution function. It assigns a probability of 0.630 at the 0.50 fractile level. The extended Swanson-Megill approximation assigns probabilities of 0.30 at both the 0.10 and 0.90 fractile levels of the cumulative probability distribution function. It assigns a probability of 0.40 at the 0.50 fractile level.

For example, we can use the extended Pearson-Tukey approximation to approximate the expected value of the cumulative probability distribution function in Fig. 4. First determine the 0.05, 0.50, and 0.95 fractiles and then assign probabilities as follows:

- 0.05 fractile is 5 with 0.185 probability
- 0.50 fractile is 30 with 0.63 probability
- 0.95 fractile is 65 with 0.185 probability

Calculate the expected value as follows:

$$EV = (5)(0.185) + (30)(0.63) + (65)(0.185)$$
$$EV = 31.85 \tag{6}$$

Decision analysis with uncertainty can also consider the risk tolerance of the decision maker. Individual decision makers can be risk adverse, risk seekers, or risk neutral. Use utility theory to consider the risk tolerance in the decision-analysis process. Assign a utility number to each possible uncertain outcome that takes into account the risk aversion of the decision maker. Expected utility replaces expected value in the analysis. Use of exponential utility functions that include a factor of risk tolerance is common. Methods for determining the level of risk tolerance use the concept of the certainty equivalent. The certainty equivalent for an uncertain alternative is the certain level of the evaluation measure that is equal in preference to the uncertain evaluation measure. Research has shown that risk tolerance often does not impact the ranking of alternatives in a decision problem [3]. We can often use expected values in place of expected utilities. This paper considers only expected values for decisions with uncertainty.

The procedure for ranking alternatives with attributes having uncertainty will be to replace the single dimensional value functions in (1) with the expected values of these functions.

The following presents an example of how to apply the above decision-analysis procedure to a protective relaying problem.

VI. Selection of Line Protection Scheme

We now apply the procedure we discussed previously in the selection process for a dual line protection scheme. As an example, a company applies two independent line protection schemes on a transmission line. The problem for the company is to decide how to best protect the transmission line with these two schemes. After much consideration, the company decides that it can meet the objectives in Table I with the solution to this decision problem. The company could use evaluation measures for all 19 objectives. However, to simplify this decision problem, we group the 19 objectives under seven categories of attributes. The seven attributes include:

- Scheme performance
- Scheme reliability
- Ease of application
- Vendor support
- Economics
- Additional benefits
- Miscellaneous

Table I
Objectives for Decision Problem

Scheme Performance
- Meets company requirements/standards for protection
- Meets protection communications requirements
- Relay operating time
- Dependability (always operates properly for line fault)
- Security (does not misoperate for remote faults)

Scheme Reliability
- Measured MTBF for relays and communication equipment
- Unavailability resulting from simultaneous common mode failure of A and B schemes

Ease of Application
- Level of company standard development necessary
- Level of engineering settings effort
- Level of training needed to implement

Vendor Support
- Overall level of technical service and support
- Quality of instruction manuals

Economics
- Overall capital cost of schemes
- Investment costs for additional spares
- O&M costs: periodic maintenance

Additional Benefits
- Ease of integrating relays into SCADA system
- Other benefits added – synchrophasors, etc.

Miscellaneous
- Company preference to apply two dissimilar schemes
- Company preference to apply relays with different hardware designs

The company selects the alternatives in Table II for evaluation, using the objectives (attributes) shown in Table I.

Table II
Alternatives for Decision Problem

Two DCB schemes, identical relays
Two DCB schemes, same manufacturer, different algorithms
Separate DCB and current differential schemes
Two DCB schemes, different manufacturers

To evaluate the alternatives on each of the seven attributes, we first construct an evaluation measure and then a single dimensional value function for each attribute.

Table III shows the scoring method used for Scheme Performance. We scored each relaying scheme separately. We gave each objective under Scheme Performance a score of zero, one, or two. The total score for Scheme Performance of a single relaying scheme is the sum of the five individual scores. We summed both evaluation scores to get an overall evaluation score for the line protection system consisting of the two separate relaying schemes.

TABLE III
SCHEME PERFORMANCE SCORING

Meets company requirements/standards for protection	Does Not [0]	Meets Most [1]	Meets All [2]
Meets company requirements/standards for communication	Does Not [0]	Meets Most [1]	Meets All [2]
Relay operating time (cycles)	> 1.5 [0]	> 1.0 & < 1.5 [1]	< 1.0 [2]
Loss of communications—fails to trip	Always [0]	Sometimes [1]	Never [2]
Loss of communications—prone to over tripping	Always [0]	Sometimes [1]	Never [2]

The total evaluation score for Scheme Performance ranges from zero to twenty. One could use a piecewise linear value function. However, with this great a range in evaluation scores, it is easier to use an exponential function to construct a value function.

We considered the midvalue to be twelve. Following the procedure in the Appendix, we calculated an exponential constant ρ of –24.32. Equation (7) and Fig. 5 show the evaluation function for Scheme Performance.

$$v_{SP}(x) = (-0.784)[1 - \exp(x/24.32)] \quad (7)$$

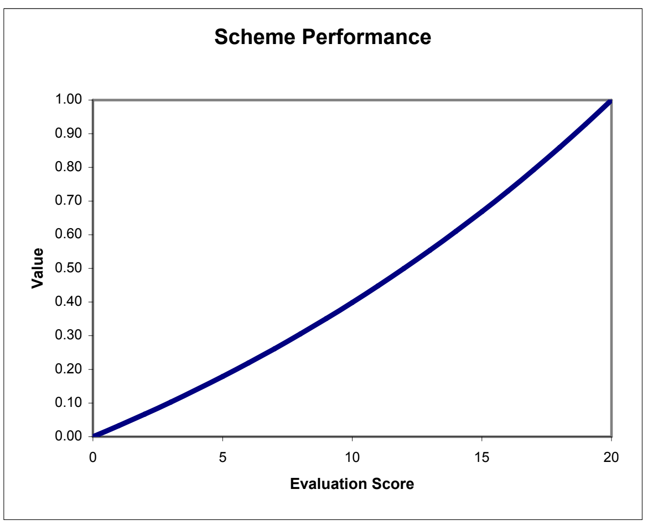

Fig. 5. Single dimensional value function for Scheme Performance

Scheme Reliability is directly associated with relay Mean Time Between Failure (MTBF). Common mode failure is the concept of multiple hardware platforms experiencing the same component failure. Table IV shows guidelines that we used in scoring this attribute. We assumed that we had obtained the MTBF from the relay manufacturers. However, it remains uncertain whether the relays in both schemes are susceptible to common mode failures. In the protective relay industry, we often have a small pool of components to select from. Relays from different manufacturers, using similar components can have different MTBFs due to component derating, the manufacturing process, and ongoing reliability testing. Table V estimates the probabilities for common mode failure. We used expected values for the evaluation score of each relay applied at a line terminal. We summed both evaluation scores to get an overall evaluation score for the line protection system consisting of the two separate relaying schemes.

TABLE IV
SCHEME RELIABILITY SCORING

Guidelines	Score
MTBF > 300 years Relays not susceptible to common mode failure	5
MTBF > 300 years Relays susceptible to common mode failure	4
200 years < MTBF < 300 years Relays not susceptible to common mode failure	3
200 years < MTBF < 300 years Relays susceptible to common mode failure	2
MTBF < 200 years Relays not susceptible to common mode failure	1
MTBF < 200 years Relays susceptible to common mode failure	0

TABLE V
PROBABILITIES OF COMMON MODE FAILURE

Condition	Probability
Relays susceptible to common mode failure	0.25
Relays not susceptible to common mode failure	0.75

The total evaluation score for Scheme Reliability ranges from zero to ten. One could use a piecewise linear value function. However, with this great a range in evaluation scores, it is easier to use an exponential function to construct a value function.

We considered the midvalue to be seven. Following the procedure in the Appendix, we calculated an exponential constant ρ of –5.55. Equation (8) and Fig. 6 show the evaluation function for Scheme Reliability.

$$v_{SR}(x) = (-0.198)[1 - \exp(x/5.55)] \quad (8)$$

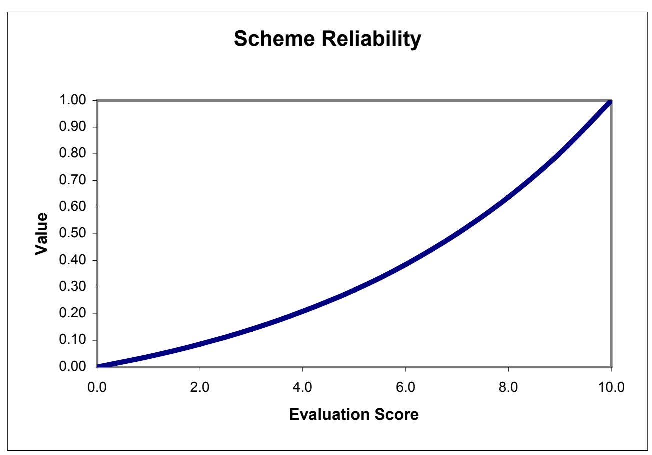

Fig. 6. Single dimensional value function for Scheme Reliability

The first step will be to determine the MTBF of each relay. For each level of MTBF in Table IV, two scores are possible.

These scores depend on the condition for common mode failure. As an example, assume that the MTBF for both relays is greater than 300 years. We can calculate the expected value for a single relay as follows:

$$EV = (0.75)(5) + (0.25)(4) = 4.75 \quad (9)$$

The overall evaluation score will be the sum of the two expected values, or 9.5 in this example. Fig. 6 uses the sum of the two expected values of the evaluation scores to show the value function for Scheme Reliability. We used the calculated expected value of 9.5 to determine a value of 0.90.

We used a piecewise linear evaluation function for the Ease of Application attribute. Table VI shows guidelines that we used in scoring this attribute.

TABLE VI
EASE OF APPLICATION SCORING

Guidelines	Score
Compliant with existing company standards Can use or duplicate existing settings templates using available software Field familiar with relay	3
Some standards modification required Engineers familiar with similar relay Field familiar with similar relay	2
Some standards modification required Engineers familiar with similar relay Field unfamiliar with relay	1
New standards development required Engineers unfamiliar with relay Field unfamiliar with relay	0

Fig. 7 shows the value function for the Ease of Application attribute. We determined that most of the increase in value results in moving from a score of 1 to a score of 2.

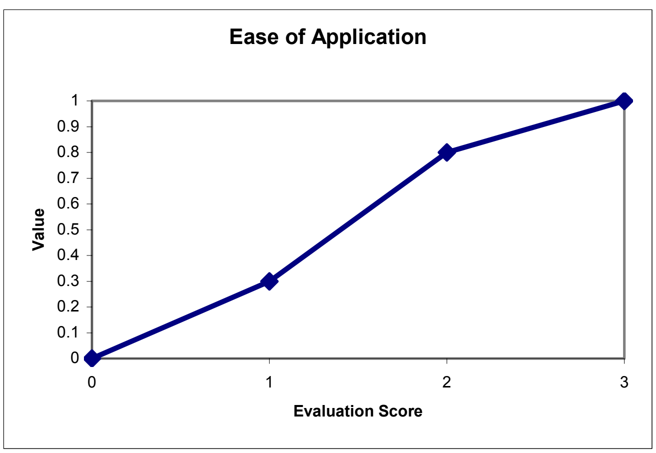

Fig. 7. Single dimensional value function for Ease of Application

We used a piecewise linear evaluation function for the Vendor Support attribute. Table VII shows guidelines that we used in scoring this attribute.

TABLE VII
VENDOR SUPPORT SCORING

Guidelines	Score
Excellent technical support & service Good instruction manuals	3
Fair technical support & service Satisfactory instruction manuals	2
Poor technical support & service Satisfactory instruction manuals	1
Poor technical support & service Poor instruction manuals	0

Fig. 8 shows the value function for the Vendor Support attribute. We determined that most of the increase in value results in moving from a score of 2 to a score of 3.

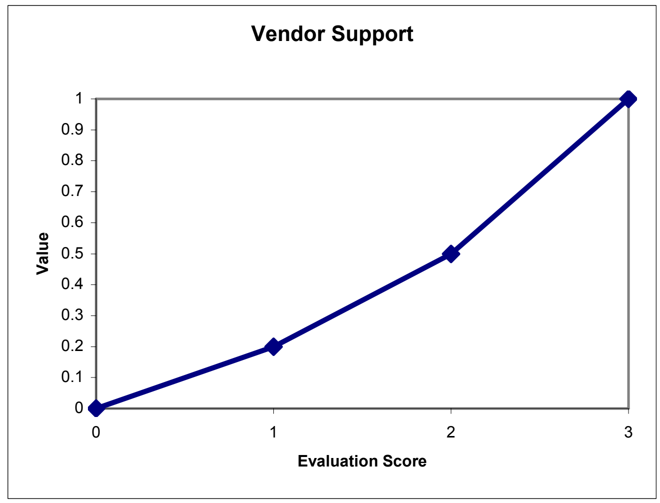

Fig. 8. Single dimensional value function for Vendor Support

The evaluation score for the Economics attribute could use the net present value (NPV) of the following costs:

- Total capital cost of both installed schemes (including relays and communication equipment)
- Capital cost plus inventory costs of any additional relays and communication equipment used as spares
- Operation and maintenance costs for periodic maintenance of both schemes

We would use the company's cost of capital in the NPV calculations, and we could consider a time period of 20 years. The value function for this case would be determined similarly as in our simplified example that follows.

For this paper, we only considered the purchase price of the four installed relays plus any spare relays. We included one spare relay for each type of relay applied, recognizing that the spare relay will be a company spare that could be used on another transmission line.

We used a decreasing exponential value function for the Economics attribute, because the evaluation score can take on an infinite number of values, and we preferred a lower purchase price. We determined for this example that the range for the purchase price of relays and spares will be from \$20k to \$90k.

We considered the midvalue to be \$35k. Following the procedure in the Appendix, we calculated exponential constant ρ of –23.28. Equation (10) and Fig. 9 show the evaluation function for Economics.

$$v_E(x) = (-0.052)[1 - \exp((90 - x)/23.28)] \tag{10}$$

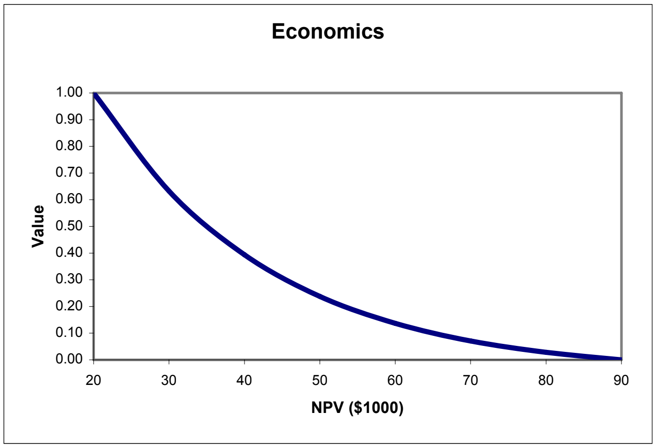

Fig. 9. Single dimensional value function for Economics

We used a piecewise linear evaluation function for the Additional Benefits attribute. Table VIII shows guidelines that we used in scoring this attribute. For the purposes of this paper, we are assuming modern digital relays will be applied with fault recording, fault locating, and metering capabilities.

TABLE VIII
ADDITIONAL BENEFITS SCORING

Guidelines	Score
Includes synchrophasors Very easy to integrate into SCADA Includes reliable peer-to-peer digital communications	3
Includes synchrophasors Some work to integrate into SCADA Includes reliable peer-to-peer digital communications	2
Includes synchrophasors Some work to integrate into SCADA Does not include reliable peer-to-peer digital communications	1
Does not include synchrophasors Some work to integrate into SCADA Does not include reliable peer-to-peer digital communications	0

Fig. 10 shows the value function for the Additional Benefits attribute. We determined that most of the increase in value results in moving from a score of 1 to a score of 2.

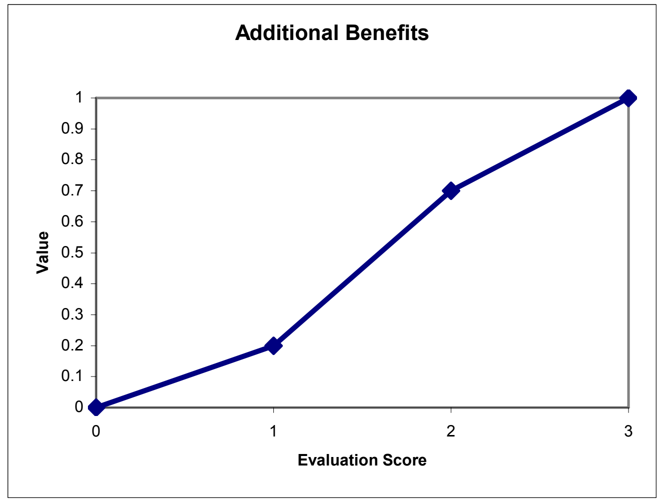

Fig. 10. Single dimensional value function for Additional Benefits

We used a piecewise linear evaluation function for the Miscellaneous attribute. Table IX shows guidelines that we used in scoring this attribute.

TABLE IX
MISCELLANEOUS SCORING

Guidelines	Score
Apply two dissimilar schemes Relays with two different hardware designs	3
Apply two dissimilar schemes Relays with same hardware design	2
Apply two similar schemes Relays with two different hardware designs	1
Apply two similar schemes Relays with same hardware design	0

Fig. 11 shows the value function for the Miscellaneous attribute. We determined that most of the increase in value results in moving from a score of 1 to a score of 2.

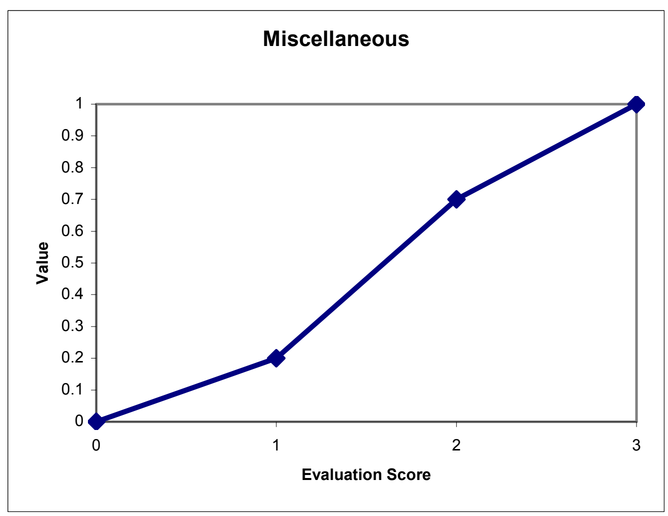

Fig. 11. Single dimensional value function for Miscellaneous

The final step before evaluating the four alternatives will be to determine the weights for each of the seven single dimensional value functions. The example that follows demonstrates this process.

We first list the seven attributes in order of increasing importance.

- Miscellaneous
- Additional Benefits
- Ease of Application
- Vendor Support
- Economics
- Scheme Reliability
- Scheme Performance

We assigned a weight of "x" to Miscellaneous and then determined the relative importance of each attribute as follows:

- Additional Benefits = 1.5 times Miscellaneous
- Ease of Application = 2 times Additional Benefits
- Vendor Support = Ease of Application
- Economics = 1.25 times Vendor Support
- Scheme Reliability = 1.5 times Economics
- Scheme Performance = 1.25 times Scheme Reliability

We can now use (2) and the relative importance of the attributes we listed previously to write an algebraic equation that can be solved. Table X lists the individual weights for each attribute.

TABLE X
CALCULATED WEIGHTS

Attribute	**Weight**
Scheme Performance	0.2823
Scheme Reliability	0.2258
Economics	0.1506
Vendor Support	0.1205
Ease of Application	0.1205
Additional Benefits	0.0602
Miscellaneous	0.0402
	1.001

We will now evaluate two of the alternatives to demonstrate the methodology described in this paper. We will consider applying two identical schemes, using Scheme A, as the first alternative. We will also consider applying two different schemes, using Scheme A and Scheme B, as the second alternative. Table XI shows the two alternatives that will be evlauated, and Table XII shows the features of the two different schemes.

TABLE XI
ALTERNATIVES EVALUATED

Alternative	Schemes Applied
1	Dual Scheme A
2	Scheme A + Scheme B

TABLE XII
SCHEME FEATURES

Scheme	Features
Scheme A	DCB scheme using Power Line Carrier
	Brand X Communication Equipment
	Cost per relay = $8,000
	MTBF of relay > 300 years
	Relay operate time < 1.0 cycle
	Software exists for developing settings templates
	Engineers familiar with relay
	Field familiar with relay
	Vendor provides excellent support & service
	Good relay instruction manuals
	Includes synchrophasors
	Very easy to integrate into SCADA
	Includes reliable peer-to-peer digital communications
Scheme B	DCB scheme using Power Line Carrier
	Brand X Communication Equipment
	Cost per relay = $10,000
	MTBF of relay < 200 years
	1.0 cycle < Relay operate time < 1.5 cycles
	Software does not exist for developing settings templates
	Engineers familiar with relay
	Field familiar with relay
	Vendor provides fair support & service
	Satisfactory instruction manuals
	Includes synchrophasors
	Some work to integrate into SCADA
	Does not Include reliable peer-to-peer digital communications

Tables XIII and XIV show how the two schemes score on Scheme Performance.

TABLE XIII
SCHEME PERFORMANCE FOR SCHEME A

Meets company requirements/standards for protection	Does Not	Meets Most	Meets All 2
Meets company requirements/standards for communication	Does Not	Meets Most	Meets All 2
Relay operating time (cycles)	> 1.5	> 1.0 & < 1.5	< 1.0 2
Loss of communications—fails to trip	Always	Sometimes	Never 2
Loss of communications—prone to over tripping	Always	Sometimes 1	Never
Total Score			**9**

TABLE XIV
SCHEME PERFORMANCE FOR SCHEME B

Meets company requirements/standards for protection	Does Not	Meets Most	Meets All 2
Meets company requirements/standards for communication	Does Not	Meets Most	Meets All 2
Relay operating time (cycles)	> 1.5	> 1.0 & < 1.5 1	< 1.0
Loss of communications—fails to trip	Always	Sometimes	Never 2
Loss of communications—prone to over tripping	Always	Sometimes 1	Never
Total Score			**8**

Alternative 1 scores a total of 18, and Alternative 2 scores a total of 17 on Scheme Performance.

We will assume the probabilities for common mode failure shown in Table V. Equation (9) calculates the expected value of the Scheme Reliability score for Scheme A. Alternative 1 will thus score twice this amount, or 9.5. Equation (11) calculates the expected value of the Scheme Reliability score for Scheme B.

$$EV = (0.75)(1) + (0.25)(0) = 0.75 \tag{11}$$

Alternative 2 will score the sum of Equation (9) and Equation (11), or 5.5.

We will assume that Alternative 1 will require five Scheme A relays, for a total cost of $40,000. Four relays will be applied in the two line protection schemes, and one relay will be the spare. We will assume that Alternative 2 will require three Scheme A relays and three Scheme B relays, for a total cost of $54,000. A spare relay will be purchased for each scheme.

Table XV shows how we score the two schemes on the remaining attributes. The total score for each alternative on these attributes will be the average of the two scores for each scheme applied. Table XVI shows the scores of the two alternatives on these attributes.

TABLE XV
SCORING ON OTHER ATTRIBUTES FOR EACH SCHEME

Attribute	Scheme A	Scheme B
Ease of application	3	2
Vendor support	3	2
Additional benefits	3	1
Miscellaneous	0	1

TABLE XVI
SCORING ON OTHER ATTRIBUTES FOR EACH ALTERNATIVE

Attribute	Alternative 1	Alternative 2
Ease of application	3	2.5
Vendor support	3	2.5
Additional benefits	3	2
Miscellaneous	0	0.5

We use the value functions defined in this paper with the above evaluation scores to calculate a value on each attribute for each alternative. We then use the weights from Table X to calculate an overall value for each alternative using Equation (1). Table XVII summarizes the results for Alternative 1, and Table XVIII summarizes the results for Alternative 2.

TABLE XVII
ALTERNATIVE 1 – EVALUATION SUMMARY

Attribute	Weight	Score	Value	Weighted Value
Scheme Performance	0.2823	18	0.86	0.243
Scheme Reliability	0.2258	9.5	0.90	0.203
Economics	0.1506	$40,000	0.39	0.059
Vendor Support	0.1205	3	1.00	0.121
Ease of Application	0.1205	3	1.00	0.121
Additional Benefits	0.0602	3	1.00	0.060
Miscellaneous	0.0402	0	0	0.000
Total Value				**0.806**

TABLE XVIII
ALTERNATIVE 2 – EVALUATION SUMMARY

Attribute	Weight	Score	Value	Weighted Value
Scheme Performance	0.2823	17	0.79	0.223
Scheme Reliability	0.2258	5.5	0.33	0.075
Economics	0.1506	$54,000	0.19	0.029
Vendor Support	0.1205	2.5	0.75	0.090
Ease of Application	0.1205	2.5	0.90	0.108
Additional Benefits	0.0602	2	0.70	0.042
Miscellaneous	0.0402	0.5	0.10	0.004
Total Value				**0.571**

Alternative 1 has a higher total value than Alternative 2. Therefore, we select Alternative 1 as the alternative that best meets all of our objectives for this line protection problem.

VII. Appendix: Calculating the Exponential Constant

The first step is to determine the midvalue of the exponential function. We define the midvalue as the evaluation score with a value of 0.5. Once we determine the midvalue, we can use Table XIX as follows to determine the exponential constant ρ.

1. Calculate the normalized midvalue by taking the difference between the midvalue and the least preferred of the two ends of the range and then dividing this total by the difference between the highest and lowest scores in the range. When doing this, take the differences so that the result is a positive number.
2. Look up the normalized midvalue in Table I under $z_{0.5}$ and find the corresponding normalized exponential constant R.
3. Determine the exponential constant ρ by multiplying R by the difference between the highest and lowest scores in the range.

TABLE XIX
CALCULATING THE EXPONENTIAL CONSTANT

$z_{0.5}$	R	$z_{0.5}$	R	$z_{0.5}$	R	$z_{0.5}$	R
0.00		0.25	0.41	0.50	Infinity	0.75	–0.410
0.01	0.014	0.26	0.435	0.51	–12.497	0.76	–0.387
0.02	0.029	0.27	0.462	0.52	–6.243	0.77	–0.365
0.03	0.043	0.28	0.491	0.53	–4.157	0.78	–0.344
0.04	0.058	0.29	0.522	0.54	–3.112	0.79	–0.324
0.05	0.072	0.30	0.555	0.55	–2.483	0.80	–0.305
0.06	0.087	0.31	0.592	0.56	–2.063	0.81	–0.287
0.07	0.101	0.32	0.632	0.57	–1.762	0.82	–0.269
0.08	0.115	0.33	0.677	0.58	–1.536	0.83	–0.252
0.09	0.130	0.34	0.726	0.59	–1.359	0.84	–0.236
0.10	0.144	0.35	0.782	0.60	–1.216	0.85	–0.220
0.11	0.159	0.36	0.845	0.61	–1.099	0.86	–0.204
0.12	0.174	0.37	0.917	0.62	–1.001	0.87	–0.189
0.13	0.189	0.38	1.001	0.63	–0.917	0.88	–0.174
0.14	0.204	0.39	1.099	0.64	–0.845	0.89	–0.159
0.15	0.220	0.40	1.216	0.65	–0.782	0.90	–0.144
0.16	0.236	0.41	1.359	0.66	–0.726	0.91	–0.130
0.17	0.252	0.42	1.536	0.67	–0.677	0.92	–0.115
0.18	0.269	0.43	1.762	0.68	–0.632	0.93	–0.101
0.19	0.287	0.44	2.063	0.69	–0.592	0.94	–0.087
0.20	0.305	0.45	2.483	0.70	–0.555	0.95	–0.072
0.21	0.324	0.46	3.112	0.71	–0.522	0.96	–0.058
0.22	0.344	0.47	4.157	0.72	–0.491	0.97	–0.043
0.23	0.365	0.48	6.243	0.73	–0.462	0.98	–0.029
0.24	0.387	0.49	12.497	0.74	–0.435	0.99	–0.014

Table XI presents pairs of numbers $z_{0.5}$ and R that solve the following equation:

$$0.5 = \frac{1 - \exp(-z_{0.5} / R)}{1 - \exp(-1 / R)} \quad (12)$$

VIII. References

[1] C. W. Kirkwood, *Strategic Decision Making: Multiobjective Decision Analysis with Spreadsheets*. Belmont, CA: Wadsworth Publishing Company, 1997.

[2] J. S. Hammond, R. L. Keeney, Howard Raiffa, *Smart Choices: A Practical Guide to Making Better Decisions*. Boston: Harvard Business School Press, 1999.

[3] R.A. Howard, "Decision Analysis: Practice and Promise," *Management Science*, vol. 34, pp. 679–695, 1988.

[4] R. L. Keeney, H. Raiffa, *Decisions with Multiple Objectives: Preferences and Value Tradeoffs*. New York: Wiley, 1976.

IX. Biography

Jon Larson is a Field Application Engineer with Schweitzer Engineering Laboratories and is located in Marshall, Michigan. Jon received BSEE and MSEE degrees from Michigan Technological University and an MBA degree from Eastern Michigan University. Jon worked fifteen years at a major Midwest utility with responsibilities primarily in protective relaying. He has been employed with Schweitzer Engineering Laboratories since October 2002. Jon is a member of IEEE and is a registered Professional Engineer in the state of Michigan.

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