Optimizing Motor Thermal Models

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OPTIMIZING MOTOR THERMAL MODELS

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ABSTRACT – The thermal limitations of induction motors are specified by thermal limit curves that are plots of the limiting temperature of the rotor and stator in units of I^2t. This paper discusses the thermal protection provided by rotor and stator thermal models defined by the thermal limit curves and supporting motor data. The thermal model is the time-discrete form of the differential equation for temperature rise due to current and is derived from fundamental principles as shown in the Appendix. The rotor model derives the slip-dependent I^2r watts using voltage and current that permit the safe starting of high-inertia drive motors. The performance of the models is shown in constant and cyclic load tests.

Index Terms: Motor Protection, Thermal Protection, Thermal Models.

I. INTRODUCTION

Protection engineers are quite familiar with the coordination of overcurrent relays for fault protection. Induction motors are thermally limited and also require thermal protection. This paper discusses thermal models that are used to determine and monitor motor temperature to prevent overheating during starting and running conditions. The thermal model is the time-discrete form of the differential equation for temperature rise due to current in a conductor. The model is derived from fundamental principles shown in the Appendix. The model relies on parameters that are defined by motor data. The derivation shows that the model can be visualized as an electric analog circuit and that the temperature can be expressed in units of I^2t. Manufacturers specify the thermal limitation using thermal limit curves that are I^2t plots of the limiting temperature. The curves for a 7000-hp, 900-rpm motor are shown in Fig. 1.

The curves represent two initial conditions: the machine initially at ambient temperature and the machine initially at operating temperature.

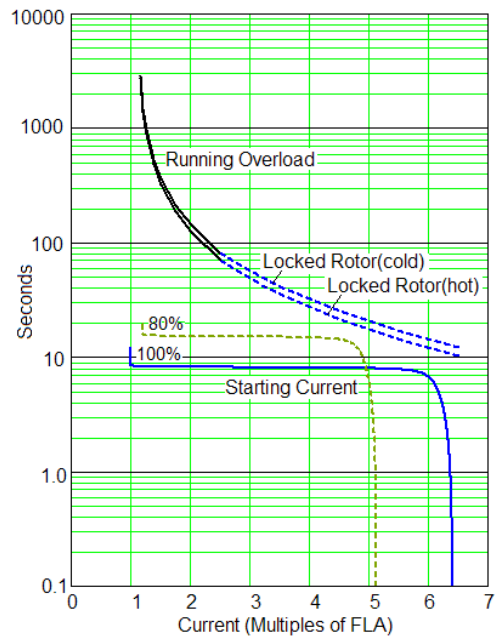

Fig. 1 7000-hp, 900-rpm Motor Thermal Limit Curves

The thermal limit curves show only two of the possible conditions of a first-order thermal process, where a balance of heat storage and heat loss determine temperature.

It is apparent that the simple dynamics of an overcurrent relay cannot provide adequate thermal protection for a motor for all operating conditions. Consequently, we will analyze the ability of the microprocessor-based thermal models to provide optimum thermal protection.

To do this, we will compare the performance of a thermal model that ignores heat loss with one that considers heat loss when applied to the same motor.

II. ADIABATIC PRINCIPLE

The paper by Lance Grainger and Michael C. McDonald, “Increasing Refinery Production by Using Motor Thermal Capacity for Protection and Control” [2] shows the derivation of a relay thermal model. Here the authors state, “Regardless of where the heating occurs, due to its rapidity, the motor can be considered an adiabatic system which absorbs energy from the equivalent stator current, but does not give off heat. Under these idealistic assumptions the temperature of the motor will increase as it absorbs energy over time.”

Then for adiabatic heating:

$$\int q \cdot dt = R\int i_{eq}^2 dt = c\omega\theta \tag{1}$$

where: C is the specific heat of the winding
ω is the weight of the conductor
θ is the temperature of the winding
R is the electrical resistance
i_{eq} is current adjusted for unbalance
q is the heat flow

From this basic relation:

$$\theta = \frac{R}{c\omega}\int i_{eq}^2\,dt \quad (2)$$

For constant current:

$$\theta = \frac{R}{c\omega} i_{eq}^2 t \quad (3)$$

Consequently, the time-current curve for a maximum temperature θ_{max} is a simple I^2t relation where $k = (\theta_{max}c\omega/R)$:

$$t(I) = \frac{k}{i_{eq}^2} \quad (4)$$

The authors state that if current is sampled periodically over some interval of time Δt, then the time to damage the motor can be calculated from the following relationship (provided the i_{eq} is greater than I_{FL}).

$$\theta_{n+1} = \theta_n + \frac{\Delta t}{t(I)} \quad (5)$$

For cooling while the motor is running, $i_{eq} < I_{FL}$, the decaying temperature is:

$$\theta_{n+1} = \theta_{FLC} + (\theta_n - \theta_{FLC})\cdot e^{-\frac{t}{\tau}} \quad (6)$$

The temperature θ_{n+1} in (5) and TC_{n+1} in (8) represents the rise above normal ambient. Consequently, θ_{FLC} and TC_{FLC} are zero for a motor at ambient. In the relay, θ_{n+1} is called the thermal capacity TC_{n+1} expressed in percent of the trip value. The equation given in the literature is:

For cooling while the motor is running, $i_{eq} < I_{FL}$

$$TC_{n+1} = TC_{FLC} + (TC_n - TC_{FLC})\cdot e^{-\frac{t}{\tau}} \quad (7)$$

For heating while the motor is running, $i_{eq} > I_{FL}$:

$$TC_{n+1} = TC_n + \left(100\cdot\frac{\Delta t}{t(I)}\right) \quad (8)$$

Equation (4) is not really the time-current curve used. If it were, $1/t(I)$ would equal I^2/k, and (6) or (8) would be simply the integration of the current squared, and any value of current greater than zero would eventually produce a trip.

If (4) is not used, what is the time-current curve? A clue is given by the parenthetical phrase "provided i_{eq} is greater than I_{FL}" quoted above. The points of the actual time-current curve are listed in the relay instruction manual, where 30 points are listed for each of 15 curves. Analysis shows that the points are an exact fit of the equation:

$$t = TM\frac{87.4}{I^2 - 1} \quad (9)$$

Where: t is the trip time in seconds
I is the current in per unit of FLA
TM is a time multiplier (integers 1 to 15)

The time-current curve, shown in Fig. 2 for TM = 1, is consistent in that it has no response for current below I_{FL}. However, (9) inserted in (5) or (8) as the t(I) implements the dynamics of an overcurrent relay rather than that of a thermal model. See IEEE C37.112 - 1996, IEEE Standard Inverse-Time Characteristic Equations for Overcurrent Relays, [3], page 4.

Consequently, this derivation produced an overcurrent model that cannot calculate temperature and will trip for cyclic overloads that do not overheat the motor.

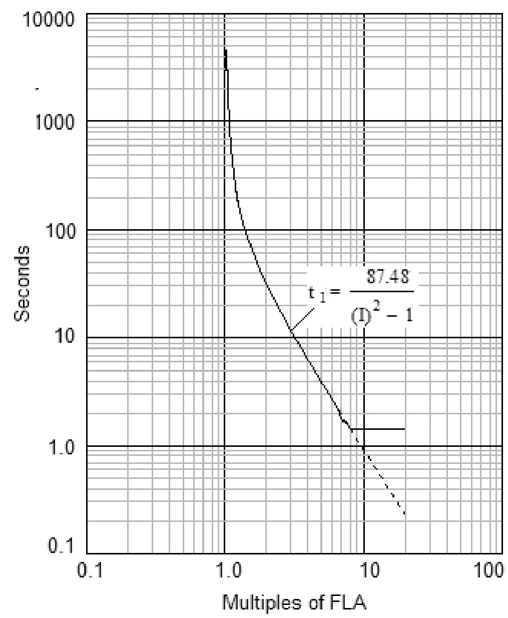

Fig. 2 Time-Current Curve for Adiabatic Derivation

III. THE EQUIVALENT CIRCUIT OF THE INDUCTION MOTOR

The sources of motor heating are the watts loss in the resistance of the rotor and stator winding. The resistances are shown in the Steinmetz model of the motor in Fig. 3. R_S is the resistance of the stator winding. R_r is the resistance of the rotor and is slip-dependent and decreases from a high locked-rotor value to a low running value at rated speed.

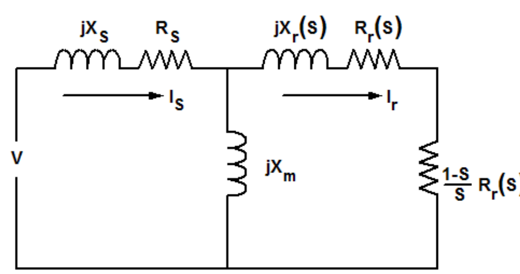

Fig. 3 Steinmetz Motor Equivalent Circuit

The positive- and negative-sequence rotor resistances are given by the linear functions of slip S:

$$R_1 = (R_M - R_N)S + R_N \quad (10)$$

$$R_2 = (R_M - R_N)(2 - S) + R_N \quad (11)$$

where: R_M is the rotor resistance at locked rotor

R_N is the rotor resistance at rated speed

R_M and R_N are known quantities defined by locked rotor current (I_L), torque (LRQ), synchronous ω_{syn}, and rated speed ω_{rated} as follows.

In the Steinmetz model shown in Fig. 3, the I^2r watts loss to the rotor resistor $(1\text{-}S)R_r/S$ is the mechanical power. Also power P_M divided by the speed $\omega = 1–S$ equals torque Q_M. Therefore:

$$Q_M = \frac{P_M}{\omega} = \frac{P_M}{1-S} = I^2 \frac{1-S}{S} R_r \frac{1}{1-S} = \frac{I^2 R_r}{S} \quad (13)$$

Solving for R_r in terms of torque, current, and slip gives:

$$R_r = \frac{Q_M}{I^2} S \quad (14)$$

For locked rotor S = 1, Q_M = LRQ

$$R_r = R_M = \frac{LRQ}{I_L^2} \quad (15)$$

The S at rated load is S_N, Current I = 1 pu, and Torque Q_M = 1 pu

$$R_N = S_N \quad (16)$$

$$R_N = \frac{\omega_{syn} - \omega_{rated}}{\omega_{syn}} \quad (17)$$

$$R_M = \frac{LRQ}{I_L^2} \quad (18)$$

The rotor resistance for any value of slip can be calculated using values taken from the plot of current and torque shown in Fig. 4.

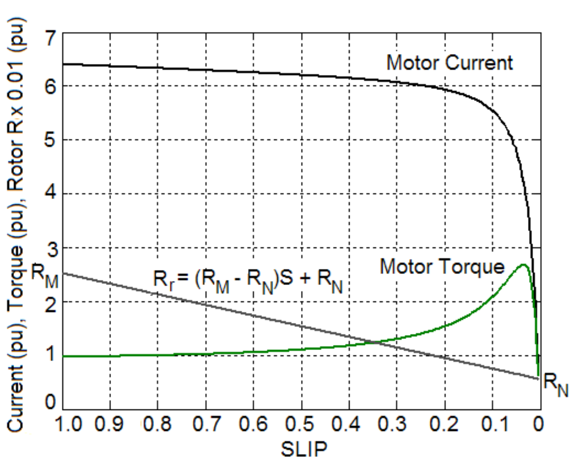

Fig. 4 Motor Current, Torque, and Rotor R Plotted Versus Slip

IV. ROTOR THERMAL MODEL

Fig. 4 shows the excessively high current drawn until the peak torque drives the motor to full speed. A starting current of six times rated current and a locked rotor resistance R_M of three times R_N causes the I^2t heating of 6^2x3 =108 times the heating of normal rated load current. Consequently, the extreme temperature caused by the high starting current must be tolerated for a limited time to allow the motor to start.

The safe starting time is indicated by the locked rotor curves shown by the dashed plot in Fig. 1. The cold locked rotor characteristic specifies the time it takes the staring current to heat the rotor to the limiting temperature with the motor initially at ambient. The hot locked rotor characteristic specifies the time for starting current to heat the rotor to the limiting temperature with the motor initially at operating temperature. The limiting temperature in units of I^2t is:

$$U_L = I_L^2 T_A \quad (19)$$

where: U_L is Rotor Temperature Limit

I_L is Locked Rotor Current in per unit of FLA

T_A is Safe stall time from ambient

Since both the hot and cold characteristic represent the same limiting temperature, the operating temperature can be expressed in terms of the limiting temperature as follows:

$$U_L = I_L^2 T_O + U_O \quad (20)$$

$$U_O = I_L^2 (T_A - T_O) \quad (21)$$

where: U_O is the operating temperature in I^2t

T_O is the safe stall time from operating temperature

Fig. 5 shows the first order thermal model that incorporates the I^2t properties of the rotor thermal limit curves as well as the effect of the slip-dependent positive- and negative-sequence rotor resistance on the input watts. The I^2t value of the operating temperature is used as the thermal resistance to ensure that one per unit input produces the operating temperature.

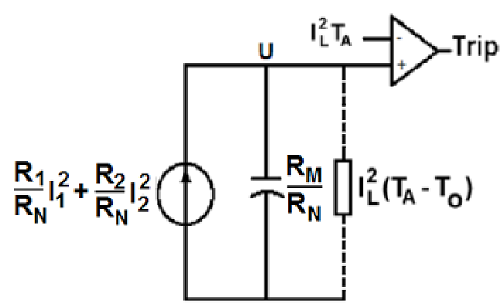

Fig. 5 Rotor Thermal Model

The following discrete form of the differential equation of the rotor thermal model is processed each sample period to calculate the temperature U:

For I > 2.5

$$U_n = \left(\frac{R_1}{R_N} I_1^2 + \frac{R_2}{R_N} I_2^2 \right) \frac{\Delta t}{C_{Th}} + U_{n-1} \quad (22)$$

For I <= 2.5

$$U_n = \left(\frac{R_1}{R_N} I_1^2 + \frac{R_2}{R_N} I_2^2\right)\frac{\Delta t}{C_{Th}} + \left(1 - \frac{\Delta t}{R_{Th} C_{Th}}\right) \cdot U_{n-1} \quad (23)$$

where the thermal capacitance $C_{Th} = R_M/R_N$ and the thermal resistance $R_{Th} = (I_L)^2(T_A – T_O)$. I_1 and I_2 are the positive- and negative-sequence currents, respectively. Note that the thermal resistance is only considered when the current drops below 2.5 pu, so that the calculation of temperature is adiabatic for starting current. At each sample, U_n is compared to the trip threshold and asserts the trip signal if the limiting temperature is exceeded.

Examples of the temperature U obtained from the rotor thermal model are shown in Figs. 6a and 6b. Note that the temperature is plotted in per unit of the limiting temperature U_L. Fig. 6a shows the locked rotor condition where R_r remains at its maximum value, and the I^2t temperature reaches the trip level in locked rotor time. Fig. 6b shows the successful start where R_r decreases, and the temperature reaches only 72% of the limiting temperature.

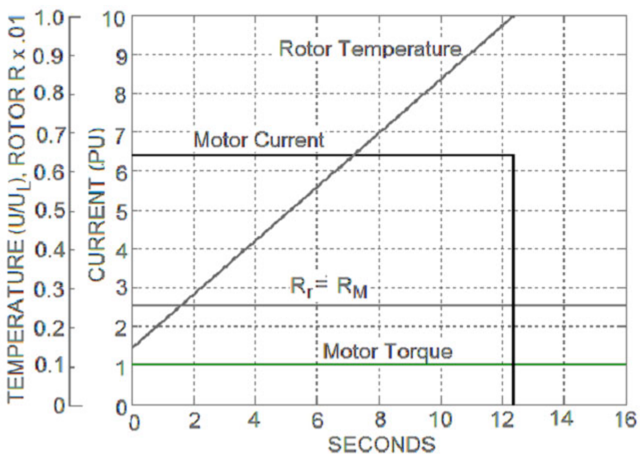

Fig. 6a Locked Rotor Trip. At Locked Rotor S=1, $R_r = R_M$

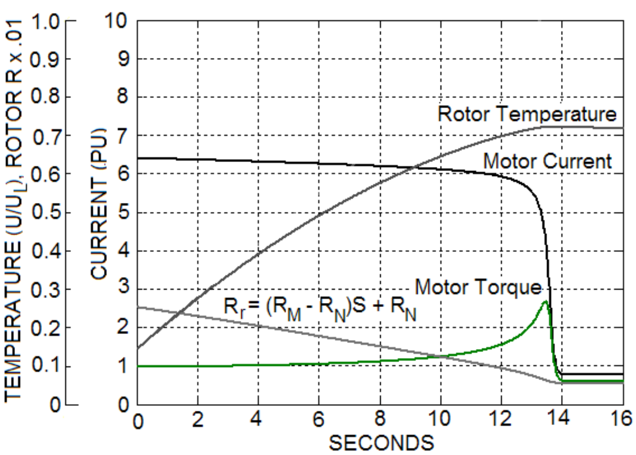

Fig. 6b Motor Starting Current, Motor Torque, R_r, and Temperature

V. CALCULATING SLIP

If the thermal model used a fixed rotor resistance R_M it would produce an I^2t rise that overestimates the temperature during valid start. This is the cause of premature tripping when starting a high-inertia motor as shown in Fig. 7. The figure shows the I^2t response of the relay reaching the trip threshold before the motor reaches running speed and the starting current subsides. The rotor reaches only 72% of the limiting temperature while the I^2t relay trips.

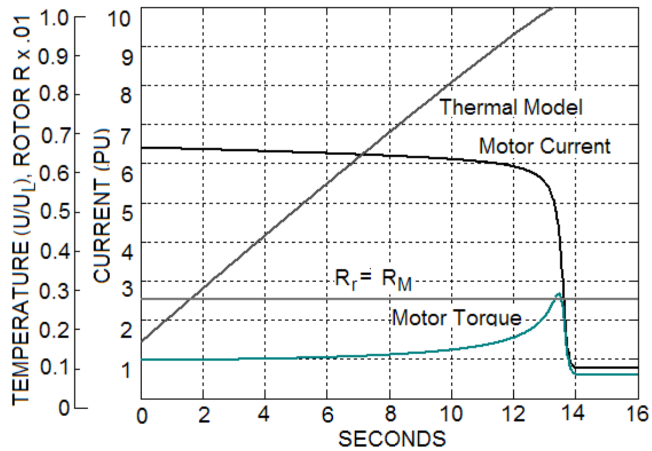

Fig. 7 Motor Starting Current With I^2t Temperature Response

To remedy this deficiency, the relay can use the measurement of voltage and current to calculate slip S. The slip can then be used to determine the slip-dependent rotor resistance. When motor voltage and current are monitored, the apparent positive-sequence impedance looking into the motor terminal is:

$$Z = R + jX = \frac{V_1}{I_1} \quad (24)$$

From the Steinmetz equivalent circuit:

$$Z = R_S + jX_S + \frac{\left(\frac{R_r}{S} + jX_r\right) \cdot jX_m}{\frac{R_r}{S} + jX_r + jX_m} \quad (25)$$

Expanding the equation:

$$Z = R_S + jX_S + \frac{\frac{R_r}{S} X_m^2 + j\left(X_m\left(\frac{R_r}{S}\right)^2 + X_r X_m (X_r + X_m)\right)}{\left(\frac{R_r}{S}\right)^2 + (X_r + X_m)^2}$$

The real part of Z is:

$$R = R_S + \frac{\frac{R_r}{S} \cdot X_m^2}{\left(\frac{R_r}{S}\right)^2 + (X_r + X_m)^2}$$

Dividing by $(X_m)^2$

$$R = R_S + \frac{\frac{R_r}{S}}{\left(\frac{R_r}{S}\right)^2 \frac{1}{X_m^2} + \frac{(X_r + X_m)^2}{X_m^2}}$$

But $\left(\frac{R_r}{S}\right)^2 \frac{1}{X_m^2}$ is negligible

Let $$A = \left(\frac{X_r + X_m}{X_m}\right)^2 \quad (26)$$

Using the real part of the motor impedance

$$R = R_S + \frac{R_r}{A \cdot S} \quad (27)$$

Substitute (10) for R_r in (27) and solve for slip S in terms of R_M, R_N and the measured resistance R.

$$S = \frac{R_N}{A(R - R_S) - (R_M - R_N)} \quad (28)$$

The slip is then used in the positive- and negative-sequence resistance equations (10) and (11). The resistance of the rotor thermal model will then be slip-dependent and produces the slip-dependent temperature rise shown in Fig. 8.

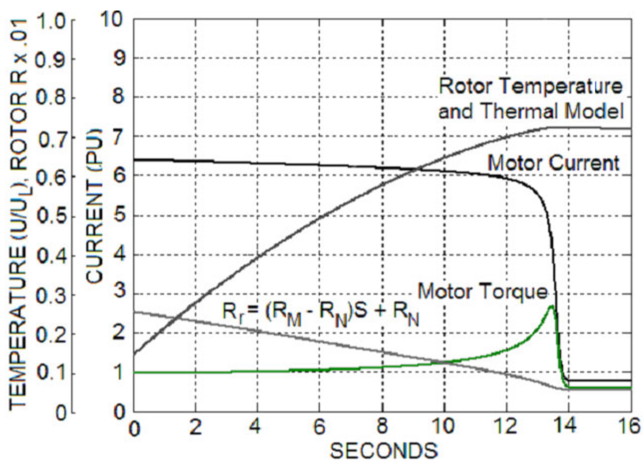

Fig. 8 Motor Starting Showing Thermal Model Emulating the Rotor Temperature

VI. OVERLOAD HEATING

The overload curves in Fig. 1 show the thermal limit of the stator. The curves fit the time-current equation:

$$t = \tau \cdot \ln\left(\frac{I^2 - I_0^2}{I^2 - SF^2}\right) \quad (29)$$

where : τ is the stator thermal time constant

I is the stator current pu of FLA

I_0 is the initial current in pu of FLA

SF is the motor service factor

The overload curves, drawn with solid lines in Fig. 1, are asymptotic to the current equal to the service factor that heats the stator to its temperature limit and is taken as the trip threshold. The stator thermal time constant can be determined by a heat run, where a load current is applied and the rise is measured at regular time intervals. The temperature will rise exponentially, and the thermal time constant will be the time it takes the temperature to reach 63.2% of its final value. In the case of the 7000-hp motor, the manufacturer listed the time constant τ as 950 seconds.

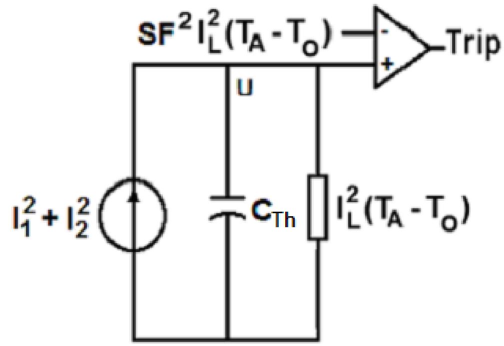

Fig. 9 Stator Thermal Model

The stator thermal model is shown in Fig. 9. $I^2(T_A - T_O)$, the operating temperature in I^2t, is retained as the thermal resistance R_{Th} since 1.0 pu current flowing in the thermal resistance produces rated operating temperature. Consequently, the trip threshold is SF^2 times the operating temperature. The thermal capacitance C_{Th} is therefore:

$$C_{Th} = \frac{\tau}{I_L^2(T_A - T_O)} \quad (30)$$

The following discrete form of the differential equation of the stator thermal model is processed each sample period to calculate the temperature U:

$$U_n = \left(I_1^2 + I_2^2\right) \cdot R_{Th} \frac{\Delta t}{C_{Th}} + \left(1 - \frac{\Delta t}{R_{Th} C_{Th}}\right) \cdot U_{n-1} \quad (31)$$

VII. STATOR TIME CONSTANT

The time-current in (29) gives the time it takes a steady-state current to raise the stator temperature to the trip level starting from the temperature caused by the previous load current I_0. Note that position of the overload curve is determined by the value of I_0 for which the manufacturer chooses to plot the curve. In Fig. 1, the initial values were chosen so that the overload curves appear to form a continuous curve with the hot and cold locked rotor curves. As plotted, these curves appear to allow the close coordination of the extremely inverse time-current overcurrent relay characteristic given by (9). However, these curves show only two of the many possible initial conditions. Consequently, the first order thermal model, as implemented in the Appendix, conserves the thermal history and provides the continuous real-time calculation of the temperature.

The time constant τ is the key parameter of the stator thermal model. When not specified, a reasonable estimate can be made as follows. Assume the motor had a previous load of 0.9 pu current when the motor is started. Where both stator and rotor are heating, under a locked rotor condition, we would expect the rotor to trip before the stator. To guarantee this condition, with locked rotor current, let the stator thermal model trip in the average of the hot and cold locked rotor time:

$$t = T_A = \tau \cdot \ln\left(\frac{I_L^2 - 0.9^2}{I_L^2 - SF^2}\right) \quad (32)$$

$$\tau = \frac{T_{AV}}{\ln\left(\frac{I_L^2 - 0.9^2}{I_L^2 - SF^2}\right)} \quad (33)$$

For the 7000-hp motor, I_L = 6.3, SF = 1.15, T_{Av} = (14+12)/2 = 13

$$\tau = \frac{13}{\ln\left(\frac{6.3^2 - 0.9^2}{6.3^2 - 1.15^2}\right)} = 979$$

In this case, the estimate is within 3% of the actual value.

VIII. COMPARING MODEL DYNAMICS

The adiabatic model is implemented using (7) and (8), where the function t(I) is the extremely overcurrent relay characteristic of (9). The time multiplier is selected to coordinate the curve with the hot locked rotor limit. The curve is:

$$t = 4.5 \cdot \frac{87.4}{I^2 - 1} \quad (34)$$

The coordination with the thermal limit curve characteristic of Fig. 1 is shown in Fig. 10.

In contrast, the settings for the stator and rotor thermal models are the parameters obtained from the 7000-hp motor data. For the stator model, the settings are the thermal time constant τ and the service factor SF:

τ = 950 Sec.

SF = 1.15

For rotor model:

ω_{syn}	= 900 rpm	Syn. Speed
ω_{rated}	= 895 rpm	Rated Speed
I_L	= 6.3 pu	Locked Rotor Current
LRQ	= 1.0 pu	Locked Rotor Torque
T_A	= 14.0 Sec.	Cold Rotor Limit
T_O	= 12.0 Sec.	Hot Rotor Limit

The relay monitors voltage and current to determine the motor Z and calculates R_M and R_N. The real part of Z is then used to derive slip and calculate the slip-dependent rotor positive- and negative-sequence resistance (see Section V).

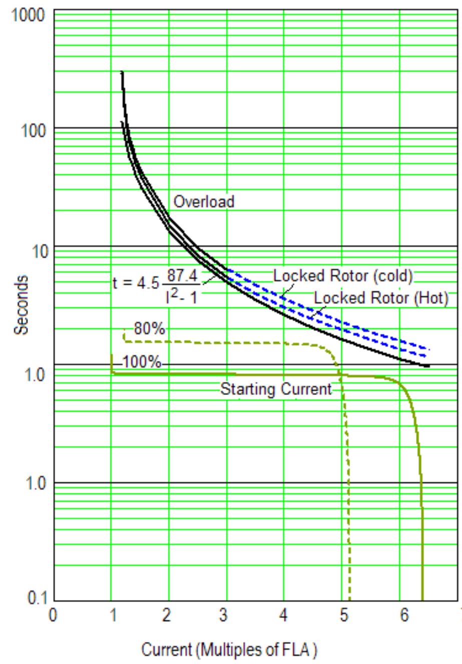

Fig. 10 Coordination of Overcurrent Relay Model With Thermal Limit Curve

$$R_N = \frac{\omega_{syn} - \omega_{rated}}{\omega_{rated}} = \frac{900 - 895}{900} = .0056$$

$$R_M = \frac{LRQ}{I_L^2} = \frac{1.0}{6.3^2} = .025$$

With these settings, the stator and thermal models take on the dynamic thermal properties of the 7000-hp motor.

A test of a thermal model is its ability to adequately protect the motor from overheating during cyclic overloads. Consequently, in this paper we will describe the results of tests in which the adiabatic model and the model considering losses were subjected to constant and cyclic overloads. The results are plotted in Figs. 11, 12, and 13.

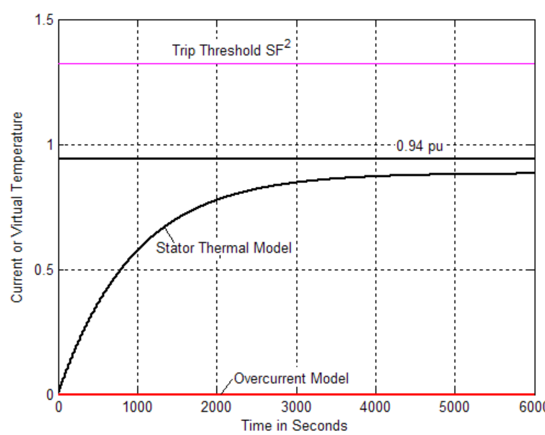

Fig. 11 Warm Up With a 0.94 pu Load Current

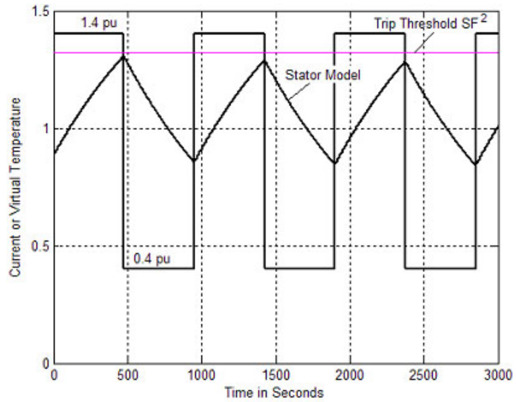

Fig. 12 Stator Model Temperature Response to Cyclic Load

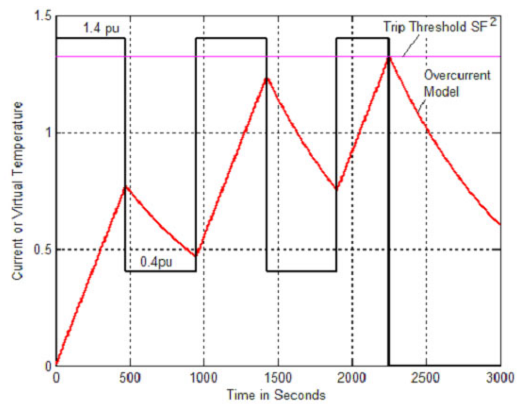

Fig. 13 Overcurrent Model Response to a Cyclic Load

In Fig. 11, the temperature calculated by the thermal model rises exponentially to a steady state value. The load current is below the pickup value of the overcurrent model, and there is no response. In Fig. 12, a maximum cycle overload is applied where the current alternated between 1.4 and 0.4 pu every 450 seconds. The cyclic load has an rms current of 1.03 pu current and does not overheat the motor. Also, the highest temperature during the cycle is short of the service factor limit. In Fig. 13, the overcurrent model trips when subjected to the cyclic overload that does not overheat the motor.

IX. CONCLUSIONS

1. The derivation by Grainger is the inadvertent implementation of an overcurrent relay with thermal reset, and implements an extremely inverse characteristic.
2. Motor thermal limit curves are plots of the limiting temperature of the rotor and stator expressed in units of I^2t.
3. Stator and rotor thermal first order models are the differential equations for heat rise in a conductor that calculates temperature rise in real time.
4. Thermal limit curves and supporting motor data define the thermal models.
5. The thermal model temperature is the square of the rms value of the cyclic current.
6. The overcurrent model trips for cyclic overload that does not overheat the motor.
7. Voltage and current monitored by a motor relay are used to calculate slip and the slip-dependent rotor resistance so that the protection of high-inertia drive motors is inherent.

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XI. APPENDIX A—THE FIRST ORDER THERMAL MODEL

The first order thermal model is derived as follows:

$$\theta = \theta_w - \theta_A \quad \text{(A1)}$$

where θ is defined as the winding temperature rise θ_W above ambient temperature θ_A

The rate of increase of the temperature is given by the equation expressing the thermal equilibrium.

$$\text{Power Supplied} - \text{Losses} = C_s m \frac{d\theta_W}{dt} = C_s m \frac{d\theta}{dt} \quad \text{(A2)}$$

In this equation, C_s is the specific heat of the winding and m is the mass. The specific heat corresponds to the amount of energy needed to raise one kilogram of that material one degree centigrade. The losses or the quantity of heat transferred to the surrounding environment is expressed as:

$$\text{Losses} = \frac{\theta_W - \theta_A}{R} = \frac{\theta}{R} \quad \text{(A3)}$$

where R is the thermal resistance in °C/Watt.

Equation (2) can be otherwise expressed as:

$$I^2 r - \frac{\theta}{R} = C_s m \frac{d\theta}{dt} \quad \text{(A4)}$$

or

$$I^2 r \cdot R = C_s m \cdot R \frac{d\theta}{dt} + \theta \quad \text{(A5)}$$

The mass m multiplied by the specific heat C_s is known as C, the thermal capacity of the system with units of joules/°C. It represents the amount of energy in joules required to raise the system temperature by one degree centigrade.

The product of the thermal resistance R and the thermal capacitance C has units of seconds and represents the thermal time constant

$$\tau = R \cdot m \cdot C_s \quad \text{(A6)}$$

The fundamental equation (5) can be expressed in a simpler form:

$$I^2 = C_s m \cdot R \cdot \left(\frac{1}{r \cdot R} \cdot \frac{d\theta}{dt} \right) + \frac{\theta}{r \cdot R} \quad \text{(A7)}$$

$$\tau = C_s m \cdot R \quad \text{(A8)}$$

let

$$U = \frac{\theta}{r \cdot R} \quad \text{(A9)}$$

and

$$\frac{dU}{dt} = \frac{1}{r \cdot R} \cdot \frac{d\theta}{dt} \quad \text{(A10)}$$

Therefore, the first order thermal model equation becomes the simple form:

$$I^2 = \tau \frac{dU}{dt} + U \quad \text{(A11)}$$

The solution of the first order equation is:

$$U = I^2 \cdot \left(1 - e^{\frac{-t}{\tau}} \right) \quad \text{(A12)}$$

With initial current I_0

$$U = I^2 \cdot \left(1 - e^{\frac{-t}{\tau}} \right) + I_0^2 \cdot e^{\frac{-t}{\tau}} \quad \text{(A13)}$$

When using (13) to calculate U over a small time increment Δt, the exponentials can be replaced with the first two terms of the infinite series as follows:

$$e^{\frac{-\Delta t}{\tau}} = \left(1 - \frac{\Delta t}{\tau} \right) \quad \text{(A14)}$$

Substituting (14) in (13) gives

$$U_n = I^2 \cdot \left(1 - \left(1 - \frac{\Delta t}{\tau} \right) \right) + U_{n-1} \cdot \left(1 - \frac{\Delta t}{\tau} \right) \quad \text{(A15)}$$

This incremental form of the equation is ideal for use in the processor for the continuous real-time calculation of temperature:

$$U_n = \frac{I^2}{\tau} \Delta t + \left(1 - \frac{\Delta t}{\tau} \right) \cdot U_{n-1} \quad \text{(A16)}$$

where U_n is the temperature expressed in units of I^2t at sample n

U_{n-1} is the temperature expressed in units of I^2t at the previous sample

Electrical engineers find it helpful to visualize the thermal model as an electrical analog circuit. The first order equation of the thermal model has the same form as the equation expressing the voltage rise in an electrical RC circuit as shown in Fig. A1.

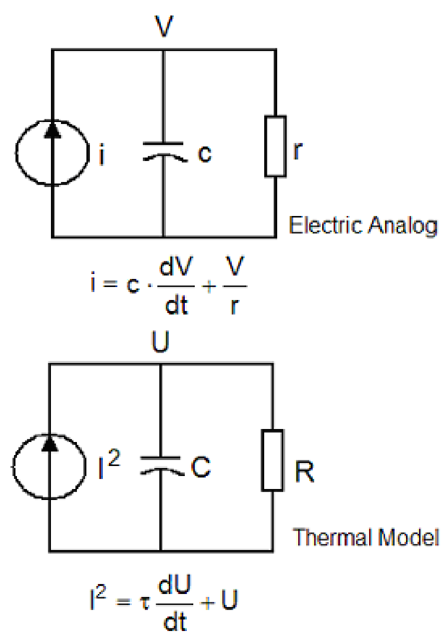

Fig. A1 The Electrical Analog Circuit of the Thermal Model

In the figure, the lower-case letters are used to identify the electrical parameters. In the circuit, the voltage V is the analog of the temperature U, the constant current i is numerically equal to the current squared. The thermal resistance R and thermal capacitance C are the direct analogs of the electrical resistance r and the electrical capacitance c.

XII. BIOGRAPHY

Stanley (Stan) Zocholl has a B.S. and M.S. in Electrical Engineering from Drexel University. He is an IEEE Life Fellow and a member of the Power Engineering Society and the Industrial Application Society. He is also a member of the Power System Relaying Committee. He joined Schweitzer Engineering Laboratories in 1991 in the position of Distinguished Engineer. He was with ABB Power T&D Company Allentown (formerly ITE, Gould, BBC) since 1947 where he held various engineering positions including Director of Protection Technology.

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