Comparing Motor Thermal Models

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ABSTRACT

Protection engineers are quite familiar with coordinating overcurrent relays to provide fault protection. In addition to fault protection, induction motors require thermal protection to prevent overheating during starting and running conditions. Manufacturers specify the thermal limitation using thermal limit curves. The IEEE Std 620-1996 *IEEE Guide for the Presentation of Thermal Limit Curves for Squirrel Cage Induction Machines* requires the manufacturer to provide a running overload and a locked rotor thermal limit curve. The thermal limit curves of a Reliance 400-hp, 3600-rpm motor are shown in Figure 1. The curves represent two initial conditions: the machine initially at ambient temperature and the machine initially at rated load operating temperature.

The thermal limit curves show only two of the possible conditions for a first-order thermal process in which the balance of heat storage and heat loss determine temperature. It is apparent that the simple dynamics of an overcurrent relay cannot provide adequate thermal protection for a motor. Consequently, we will analyze the ability of the microprocessor-based thermal models to provide optimum thermal protection. To do this, we will compare the performance of a thermal model that ignores heat loss with one that considers heat loss when applied to the same motor.

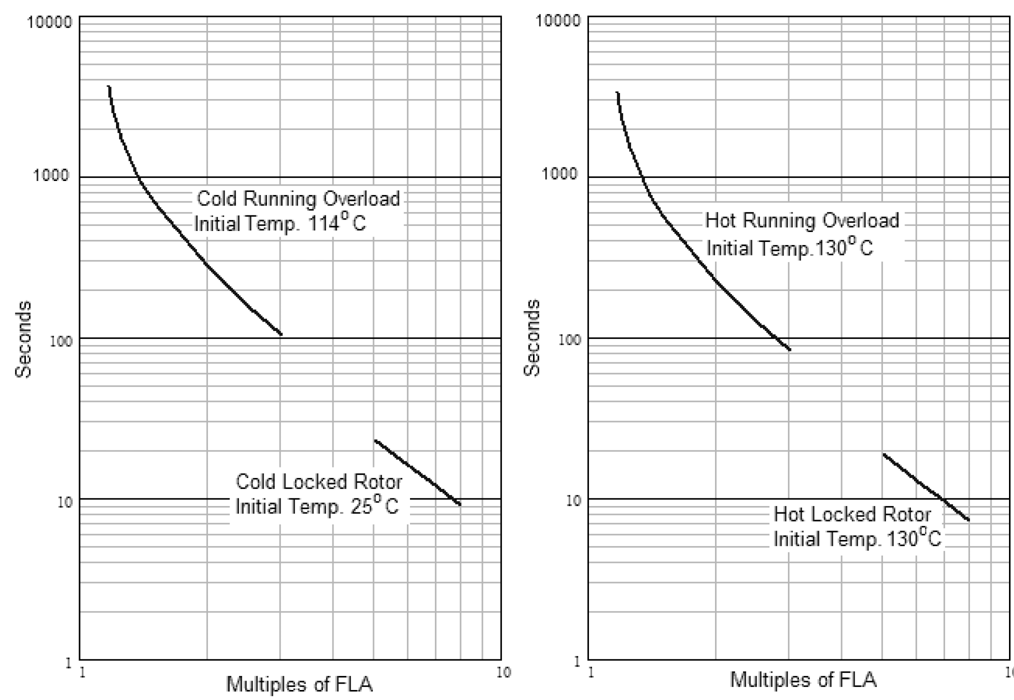

Figure 1 Thermal Limit Curves for a 400-hp, 3600-rpm Induction Motor

ADIABATIC PRINCIPLE USED IN RELAY TYPE A

The paper by Lance Grainger and Michael C. McDonald, "Increasing Refinery Production by Using Motor Thermal Capacity for Protection and Control," *IEEE Transactions on Industry Applications,* Vol. 33, No. 3, May/June 1997, page 858, shows the derivation of the Relay Type A thermal model. Here the authors state, "Regardless of where the heating occurs, due to its rapidity, the motor can be considered an adiabatic system which absorbs energy from the equivalent stator current, but does not give off heat. Under these idealistic assumptions the temperature of the motor will increase as it absorbs energy over time."

Then for adiabatic heating:

$$\int q \cdot dt = R\int i_{eq}^{2} dt = c\omega\theta \quad (1)$$

where:		
	c	is the specific heat of the motor winding
	ω	is the weight of the motor winding metal
	θ	is the temperature of the winding
	R	is the electrical resistance of the winding
	i_{eq}	is the stator current adjusted for negative sequence
	q	is the heat flow

From this basic relation:

$$\theta = \frac{R}{c\omega}\int i_{eq}^{2}\, dt \quad (2)$$

For constant current:

$$\theta = \frac{R}{c\omega} i_{eq}^{2} t \quad (3)$$

Consequently, the time-current curve for a maximum temperature θ_{max} is a simple I^2t relation where $k = (\theta_{max} c\omega/R)$:

$$t(I) = \frac{k}{i_{eq}^{2}} \quad (4)$$

The authors state that if current is sampled periodically over some interval of time, Δt, then the time to damage the motor can be calculated from the following recursive relationship (provided i_{eq} is greater than I_{FL}):

For heating while the motor is running, $i_{eq} > I_{FL,}$ the temperature θ is:

$$\theta_{n+1} = \theta_n + \frac{\Delta t}{t(I)} \quad (5)$$

For cooling while the motor is running, $i_{eq} < I_{FL}$

$$\theta_{n+1} = \theta_{FLC} + \left(\theta_n - \theta_{FLC}\right) \cdot e^{-\frac{t}{\tau}} \quad (6)$$

The temperature θ_{n+1} in Equation (5) and TC_{n+1} in Equation (8) represents the rise above normal ambient. Consequently, θ_{FLC} and TC_{FLC} are zero for a motor at ambient. In the relay, θ_{n+1} is called the thermal capacity TC_{n+1} expressed in percent of the trip value. The equation given in the literature is:

For cooling while the motor is running, $i_{eq} < I_{FL}$

$$TC_{n+1} = TC_{FLC} + \left(TC_n - TC_{FLC}\right) \cdot e^{-\frac{t}{\tau}} \tag{7}$$

For heating while the motor is running, $i_{eq} > I_{FL}$

$$TC_{n+1} = TC_n + \left(100 \cdot \frac{\Delta t}{t(I)}\right) \tag{8}$$

Equation (4) is not really the time-current curve used. If it were, 1/t(I) would equal I^2/k, and Equation (6) or (8) would be simply the integration of the current squared, and any value of current greater than zero would eventually produce a trip.

If Equation (4) is not used, what is the time-current curve? A clue is given by the parenthetical phrase "provided i_{eq} is greater than I_{FL}" quoted above. The points of the actual time-current curve are listed in the relay instruction manual, where 30 points are listed for each of 15 curves. A Mathcad® study of the points will show that the points are an exact fit of the equation:

$$t(I) = TM \cdot \left(\frac{87.48}{I^2 - 1}\right) \tag{9}$$

where:	t	is the trip time in seconds
	I	is the relay current in per unit of FLA
	TM	is a curve multiplier (integers from 1 to 15)

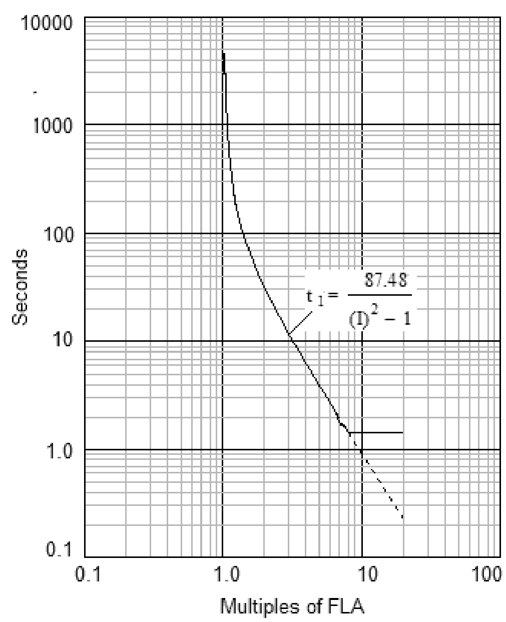

Figure 2 Curve No. 1 of Relay Type A

The time-current curve used is consistent in that it has no response for current below I_{FL}. However, Equation (9) inserted in Equation (5) or (8) as the t(I) implements the dynamics of an overcurrent relay rather than that of a thermal model. See IEEE C37.112 - 1996, *IEEE Standard Inverse-Time Characteristic Equations for Overcurrent Relays*, Equation (3), page 4.

Consequently, the Type A thermal model is an overcurrent model that cannot calculate temperature and will trip for cyclic overloads that do not overheat the motor.

FIRST ORDER MODEL USED IN RELAY TYPE B

The thermal model is described in and covered by U.S. Patent No. 5,436,784. It is a first-order thermal model derived from the following settings:

FLA	Rated full load motor current in secondary amps
LRA	Rated locked rotor current in secondary amps
LRT	Thermal limit time at rated locked rotor current
TD	Time dial to trip temperature in per unit of LRT
SF	Motor rated service factor

It is a rotor thermal model shown as an electrical analog in Figure 3 that has a starting state asserted when the current is greater than 2.5 per unit of full load current and a running state asserted when the motor current is less than 2.5 per unit of full load current. The ratio R_1/R_0 is set equal to 3.0 in the rating method.

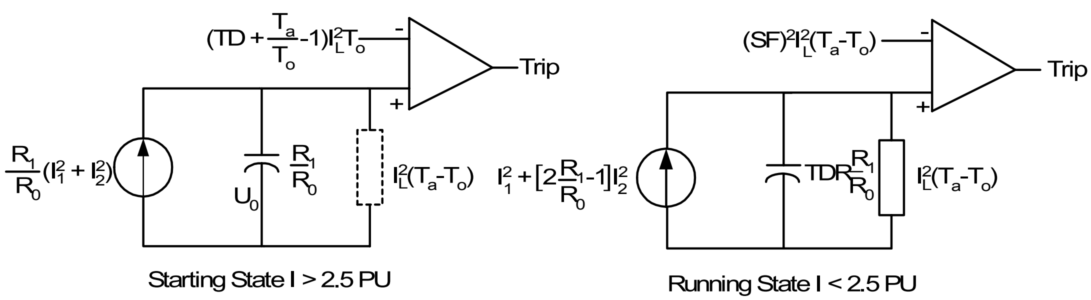

Figure 3 Thermal Model Used in Relay Type B

The parameters of the thermal model are defined as follows:

R_1	= Locked rotor electrical resistance (per unit ohms)
R_0	= Running rotor electrical resistance also rated slip (per unit ohms)
R_1/R_0	= 3
I_L	= Locked rotor current in per unit of full load current
T_a	= Locked rotor time with motor initial at ambient
T_o	= Locked rotor time with motor initially at operating temperature
TD	= Locked rotor time dial
I_1	= Positive-sequence motor current in per unit of FLA
I_2	= Negative-sequence motor current in per unit of FLA

TDR is a factor that equalizes trip times of ambient-running and hot-start curves at I_L when the running time constant setting RTC = AUTO where:

$$TDR = \frac{TD}{0.6 \cdot I_L^2 \cdot \ln\left(\frac{I_L^2}{I_L^2 - SF^2}\right)} \tag{10}$$

The user has the option of setting RTC to the time constant of the stator if the value is known.

The type B first order thermal model is implemented as follows where θ is defined as the winding temperature rise above ambient:

$$\theta = \theta_w - \theta_A \tag{11}$$

The rate of increase of the temperature is given by the equation expressing the thermal equilibrium:

$$\text{Power Supplied} - \text{Losses} = C_s m \frac{d\theta_W}{dt} = C_s m \frac{d\theta}{dt} \tag{12}$$

In this equation, C_s is the specific heat of the winding and m is the mass. The specific heat corresponds to the amount of energy needed to raise 1 kilogram of that material 1 degree centigrade. The losses or the quantity of heat transferred to the surrounding environment is expressed as:

$$\text{Losses} = \frac{\theta_W - \theta}{R} \tag{13}$$

where R is the thermal resistance in °C/Watt

Equation 12 can be otherwise expressed as:

$$I^2 r - \frac{\theta}{R} = C_s m \frac{d\theta}{dt} \tag{14}$$

or:

$$I^2 r \cdot R = C_s m \cdot R \frac{d\theta}{dt} + \theta \tag{15}$$

The mass m multiplied by the specific heat C_s is known as C, the thermal capacity of the system with units of joules/°C. It represents the amount of energy in joules required to raise the system temperature by 1 degree centigrade.

The product of the thermal resistance R and the thermal capacitance C has units of seconds and represents the thermal time constant T_{th} of the system:

$$T_{th} = R \cdot m \cdot C_s \tag{16}$$

The fundamental Equation (15) can be expressed in a simpler form:

$$I^2 = C_s m \cdot R \cdot \left(\frac{1}{r \cdot R} \cdot \frac{d\theta}{dt} \right) + \frac{\theta}{r \cdot R} \tag{17}$$

$$T_{th} = C_s m \cdot R \tag{18}$$

let
$$U = \frac{\theta}{r \cdot R} \tag{19}$$

and
$$\frac{dU}{dt} = \frac{1}{r \cdot R} \cdot \frac{d\theta}{dt} \tag{20}$$

$$I^2 = T_{th} \frac{dU}{dt} + U \tag{21}$$

where: T_{th} = thermal time constant
U = virtual temperature in units of I^2

The time-discrete form of Equation (21) can be written as:

$$I^2 = T_{th} \cdot \frac{U_n - U_{n-1}}{\Delta t} + U_{n-1} \tag{22}$$

Solving for U_n provides the following iterative equation for virtual temperature:

$$U_n = \frac{I^2}{T_{th}} \Delta t + \left(1 - \frac{\Delta t}{T_{th}} \right) \cdot U_{n-1} \tag{23}$$

where U_n is the virtual temperature at sample n
U_{n-1} is the virtual temperature at the previous sample

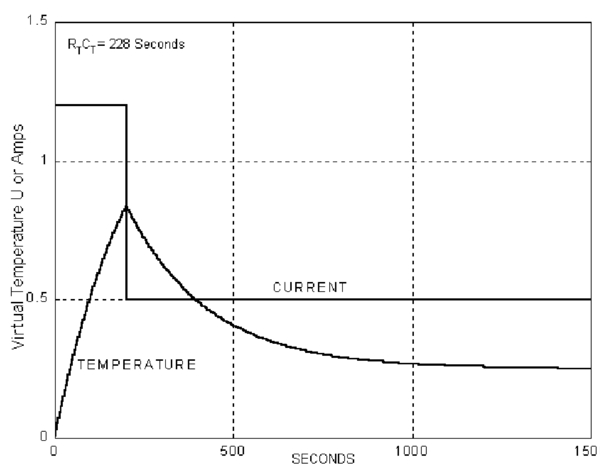

Figure 4 Dynamic Response of the First Order Thermal Model

Equation (23) is the basis of the algorithm that enables a microprocessor-based relay to continuously calculate the temperature of a thermal process. The algorithm also monitors the temperature and asserts a trip or an alarm signal when it exceeds predetermined thresholds. It should be kept in mind that when thermal protection is implemented using this equation, the cold and the hot time-limit curves are intrinsically embedded in the process. In reality, the protection responds properly, irrespective of the starting current. Figure 4 shows the dynamic response of the algorithm to a high pulse of current followed by a lower constant current. The temperature shows an exponential rise to a peak followed by an exponential decay to the final value.

COMPARING THE RELAY TYPE A AND RELAY TYPE B DYNAMIC RESPONSE

Figure 5 shows the time-current characteristics of the Type A and the Type B relays applied to the thermal limit curves of a 400-hp motor. Relay Type B is applied by entering, as settings, the full load current, the locked rotor current, the hot locked rotor time, and the service factor of the motor. Relay Type A is applied by selecting a curve to coordinate with the locked rotor limit curve. The curves indicate trip time for when a constant current is applied for the specified initial condition as would be the case for any overcurrent curve coordinated in a like manner. However, the test of a thermal model is its ability to adequately protect the motor from overheating during cyclic overloads. Consequently, in this paper we will describe the results of a series of simulations in which both relays are subjected to constant and cyclic overloads.

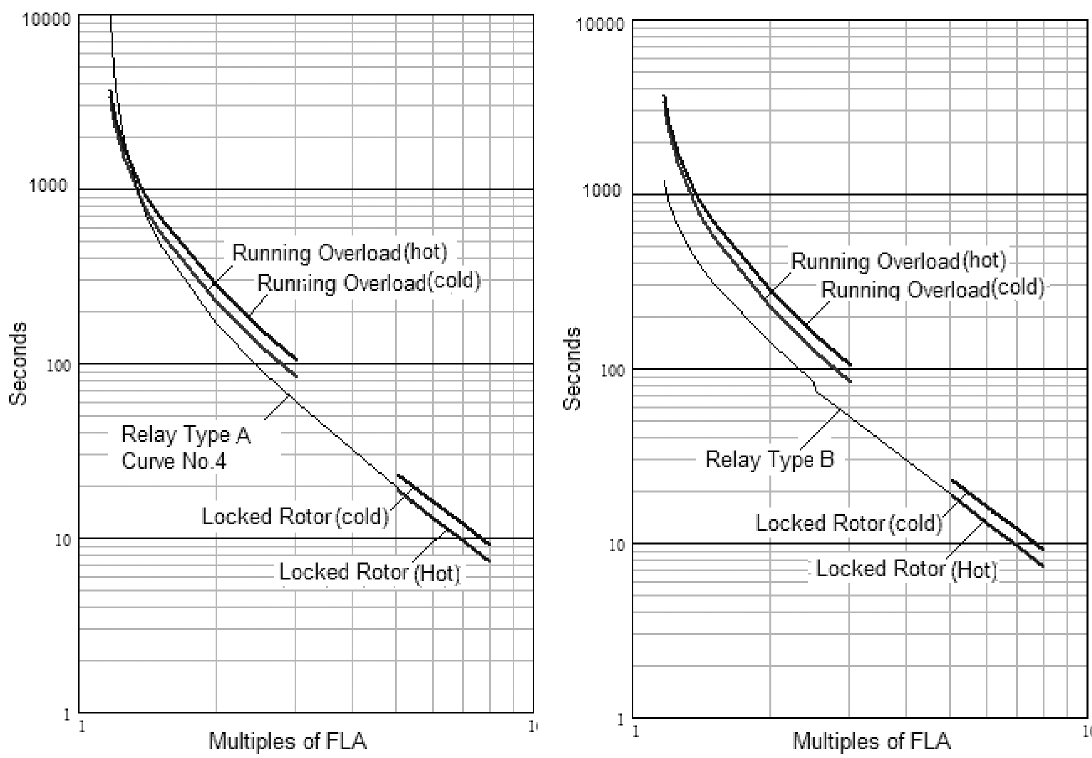

Figure 5 Time-Current Characteristics Applied to a 400-hp, 3600-rpm Motor

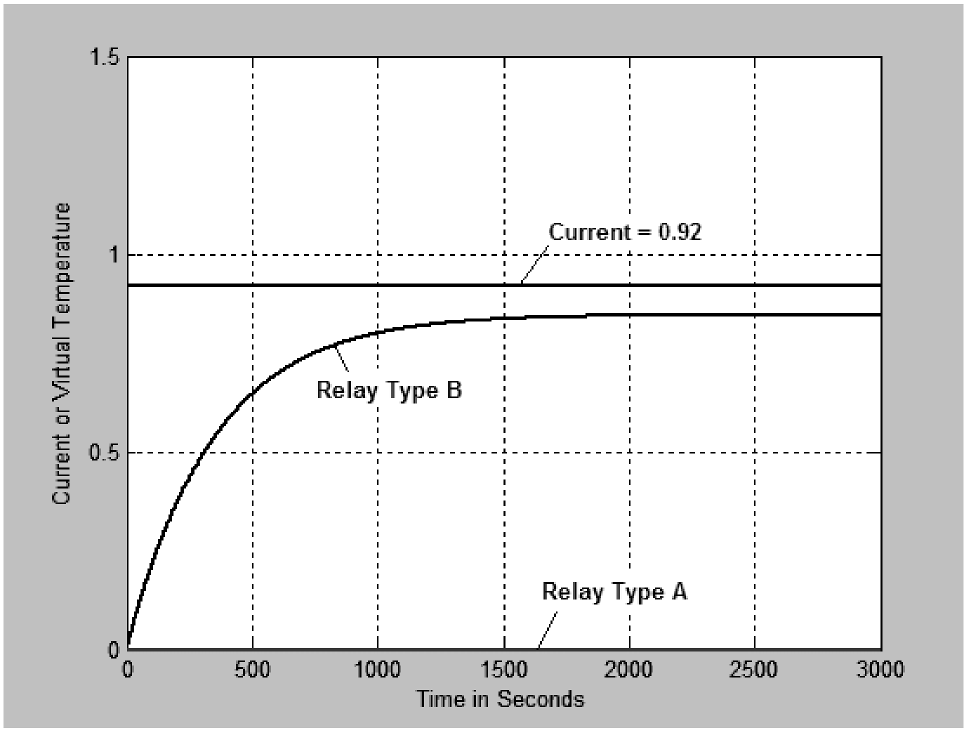

Figure 6 Responses to a Current Less Than the Service Factor

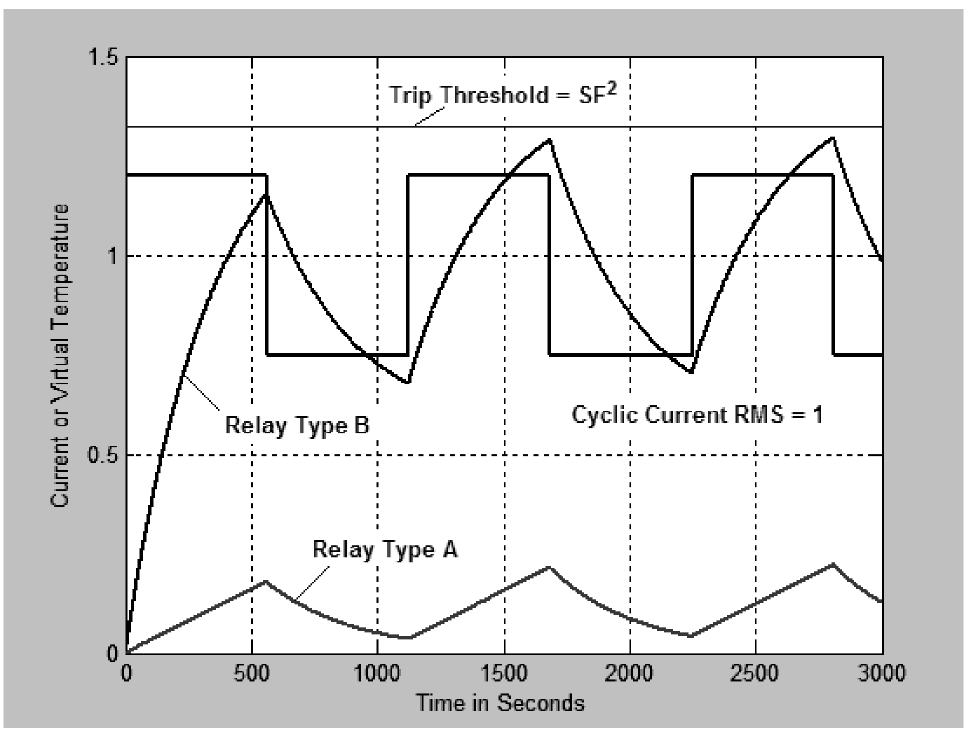

Figure 7 Response to Cyclic Overload RTC = AUTO

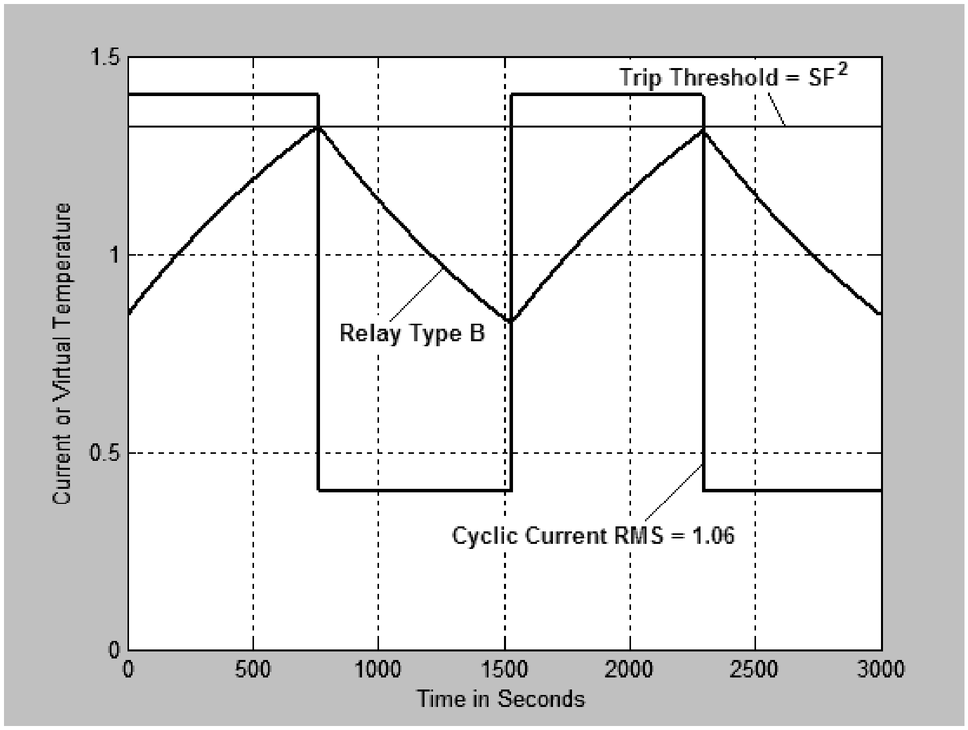

Figure 8 Response of Relay Type B to a Cyclic Current RTC = 1370

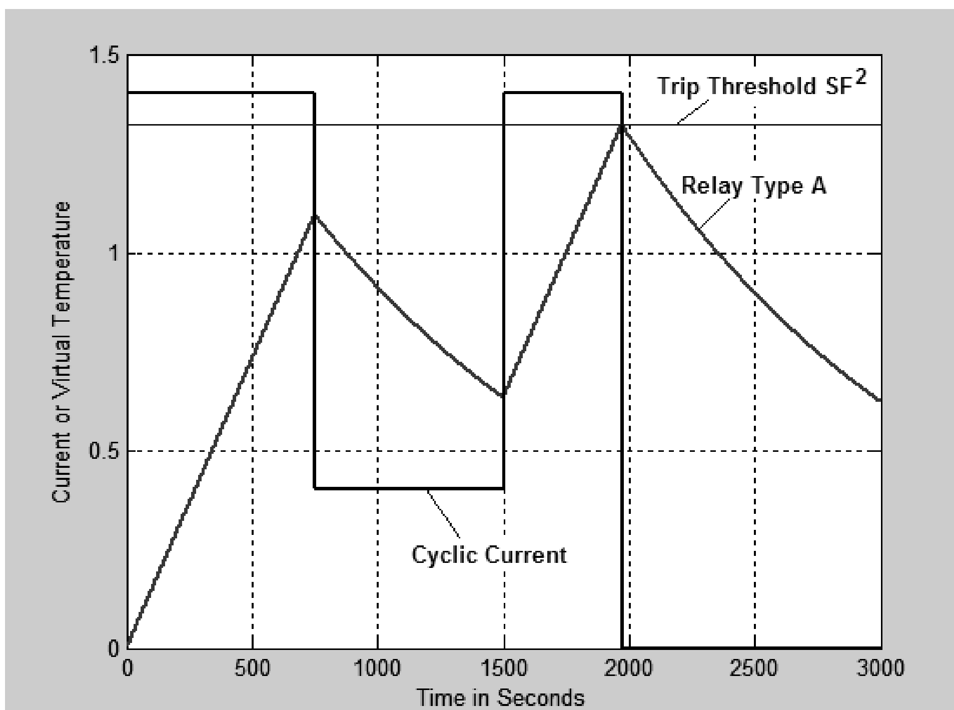

Figure 9 Premature Trip of Relay Type A to a Cyclic Current RTC = 1370

CONCLUSIONS

1. The derivation by Grainger shows that the Relay Type A is inadvertently implemented as an overcurrent relay with thermal reset and not as a thermal model.
2. Grainger implies the integration of I^2. Actually it is the integration of $(I^2 - 1)$ that produces the extremely inverse time-current characteristic $t = A/(I^2 - 1)$.
3. Relay Type B uses a first-order thermal model that is the differential equation for heat rise in a conductor that calculates temperature in real time.
4. As shown in Figure 6, the Relay Type A model cannot measure temperature and has no response to any current below the pickup current. In contrast, the Relay Type B thermal model calculates the temperature rise at all times and for any current.
5. The Relay Type B responds to cyclic current as shown in Figure 7. In this case, the current switches between 1.2 and 0.7483 per unit current every 550 seconds. The waveform has a 1.0 per unit RMS value and produces 1.0 per unit heat. In contrast, the Relay Type A hardly responds.
6. The Relay Type B thermal model provides optimum thermal protection when RTC is set to match the time constant of the 400-hp motor. Figure 8 shows the Relay Type B response to the maximum cyclic overload that can be sustained without overheating the motor.
7. Figure 9 shows that the Relay Type A causes a premature shutdown when subjected to the same cyclic current.

REFERENCES

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[4] S. E. Zocholl and G. Benmouyal, "Using Thermal Limit Curves to Define Thermal Models of Induction Motors," *Proceedings of the 28th Annual Western Protective Relay Conference*, Spokane, WA, October 23–25, 2001.

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