SYMMETRICAL COMPONENTS: LINE TRANSPOSITION

Stanley E. Zocholl
Schweitzer Engineering Laboratories, Inc.
Pullman, WA USA

LINE IMPEDANCE MATRIX

Symmetrical components are derived from the impedance matrix of a transposed transmission line. The impedance matrix is obtained by writing equations for the voltage drop due to current in each phase:

$$\begin{aligned} V_a &= Z_{aa}I_a + Z_{ab}I_b + Z_{ac}I_c \\ V_b &= Z_{ba}I_a + Z_{bb}I_b + Z_{bc}I_c \\ V_c &+ Z_{ca}I_a + Z_{cb}I_b + Z_{cc}I_c \end{aligned} \quad (1)$$

When using matrix notation, the impedance is seen as a 3-by-3 matrix of self and mutual impedance terms, and the voltage and current are seen as 1-by-3 column vectors:

$$\begin{pmatrix} V_a \\ V_b \\ V_c \end{pmatrix} = \begin{pmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{pmatrix} \begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} \quad (2)$$

Equation (2) can be written in more compact notation where bold letters indicate matrices:

$$\mathbf{V} = \mathbf{Z}\,\mathbf{I} \quad (3)$$

SYMMETRICAL COMPONENT MATRIX

The symmetrical component equations for voltage are:

$$\begin{aligned} V_{a0} &= \frac{1}{3}\left(V_a + V_b + V_c\right) \\ V_{a1} &= \frac{1}{3}\left(V_a + aV_b + a^2V_c\right) \\ V_{a2} &= \frac{1}{3}\left(V_a + a^2V_b + aV_c\right) \end{aligned} \quad (4)$$

Which when written in matrix notation appear as:

$$\begin{pmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix} \begin{pmatrix} V_a \\ V_b \\ V_c \end{pmatrix} \quad (5)$$

Using the more compact notation:

$$\mathbf{V_{sym} = A\ V} \tag{6}$$

Matrix A contains the vector 1 and the plus and minus 120° rotation unity vectors a and a2. Similarly, the symmetrical component equations for current are:

$$\begin{aligned} I_{a0} &= \frac{1}{3}\left(I_a + I_b + I_c\right) \\ I_{a1} &= \frac{1}{3}\left(I_a + aI_b + a^2 I_c\right) \\ I_{a2} &= \frac{1}{3}\left(I_a + a^2 I_b + aI_c\right) \end{aligned} \tag{7}$$

Which can be written in matrix form as:

$$\begin{pmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix} \begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} \tag{8}$$

and in more compact notation:

$$\mathbf{I_{sym} = A\ I} \tag{9}$$

Matrix equations (6) and (9) are the 1-by-3 voltage and current column vectors in terms of the symmetrical component operator **A**. The relation between the symmetrical components and the impedance matrix is obtained by substituting these equations for **V** and **I** in (3):

$$\mathbf{A\ V_{sym} = A\ Z\ I_{sym}}$$

$$\mathbf{V_{sym} = A\ Z\ A^{-1}\ I_{sym}} \tag{10}$$

Observing the position of terms in (10), we can write the symmetrical component impedance matrix as:

$$\mathbf{V_{sym} = Z_{syn}\ I_{sym}} \tag{11}$$

where the symmetrical component impedance matrix is:

$$\mathbf{Z_{syn} = A\ Z\ A^{-1}} \tag{12}$$

and where Z is the transmission line impedance matrix:

$$\mathbf{Z} = \begin{pmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{pmatrix} \tag{13}$$

In the Z matrix, the self-impedance terms are equal such that:

$$Z_s = Z_{aa} = Z_{bb} = Z_{cc} \tag{14}$$

For a transposed line, the off-diagonal terms, which are the mutual impedances between conductors, are equal such that:

$$Z_m = Z_{ab} = Z_{bc} = Z_{ca} \tag{15}$$

The impedance matrix can then be written as:

$$Z = \begin{pmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{pmatrix} \tag{16}$$

and the component matrix for the transposed transmission line is:

$$\mathbf{Z_{syn}} = \mathbf{A\ Z\ A^{-1}} = \begin{pmatrix} Z_s + 2Z_m & 0 & 0 \\ 0 & Z_s - Z_m & 0 \\ 0 & 0 & Z_s - Z_m \end{pmatrix} \tag{17}$$

In the symmetrical component matrix, the diagonal terms are the zero-, positive-, and negative-sequence impedance of the transmission line. Note that the off-diagonal terms are zero and indicate that there is no coupling between the zero-, positive-, and negative-sequence networks for the transposed line.

$$\begin{aligned} Z_0 &= Z_s + 2Z_m \\ Z_1 &= Z_s - Z_m \\ Z_2 &= Z_s - Z_m \end{aligned} \tag{18}$$

The impedance matrix for a transposed 100-mile, 500 kV, flat construction line is given in (19). The impedance is given in per unit on a 500 kV, 1000 MVA base:

$$\mathbf{Z} = \begin{pmatrix} 0.048 + j0.432 & 0.036 + j0.17 & 0.036 + j0.17 \\ 0.036 + j0.17 & 0.048 + j0.432 & 0.036 + j0.17 \\ 0.036 + j0.17 & 0.036 + j0.17 & 0.048 + j0.432 \end{pmatrix} \tag{19}$$

The symmetrical component matrix is obtained by applying equation (12):

$$\mathbf{Z_{syn}} = \mathbf{A\ Z\ A^{-1}} = \begin{pmatrix} 0.12 + j0.771 & 0 & 0 \\ 0 & 0.012 + j0.262 & 0 \\ 0 & 0 & 0.012 + j0.262 \end{pmatrix} \tag{20}$$

Where the zero-, positive-, and negative-sequence impedance terms are:

$$\begin{aligned} Z_0 &= 0.12 + j0.771 \\ Z_1 &= 0.012 + j0.262 \\ Z_2 &= 0.012 + j0.262 \end{aligned} \tag{21}$$

TRANSPOSED LINES

The three-phase fault calculations for a transposed line are as follows:

$$\begin{pmatrix} V_a \\ V_b \\ V_c \end{pmatrix} = \begin{pmatrix} 1 \\ a^2 \\ a \end{pmatrix} \qquad \begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} = Z^{-1} \begin{pmatrix} V_a \\ V_b \\ V_c \end{pmatrix} \qquad \begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} = \begin{pmatrix} 0.175 - j3.803 \\ -3.381 + j1.75 \\ 3.206 + j2.053 \end{pmatrix}$$

$$Z1a = \frac{V_a - V_b}{I_a - I_b} \qquad Z1b = \frac{V_b - V_c}{I_b - I_c} \qquad Z1c = \frac{V_c - V_a}{I_c - I_a}$$

$$\left|\frac{Z1a}{Z_{a1}}\right| = 1.00 \qquad \left|\frac{Z1b}{Z_{a1}}\right| = 1.00 \qquad \left|\frac{Z1c}{Z_{a1}}\right| = 1.00$$

The three-phase calculations using symmetrical components give:

$$\begin{pmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{pmatrix} = Z_{sym}^{-1} \begin{pmatrix} V_{a0} \\ V_{a1} \\ V_{a3} \end{pmatrix}$$

$$\begin{pmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{pmatrix} \frac{1}{I_{a1}} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$Z_{a1} = \frac{V_{a1}}{I_{a1}} \qquad \frac{\mathrm{Im}(Z_{a1})}{\mathrm{Im}(Z_{sym_{1,1}})} = 1.00$$

The calculations for the transposed line indicate an ideal mho element response. Such is not the case for a nontransposed line.

NONTRANSPOSED LINES

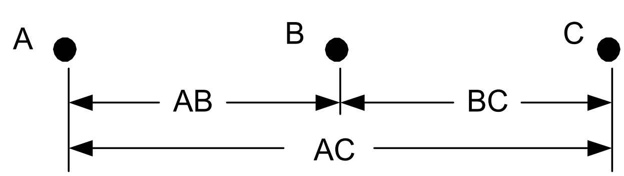

Figure 1 Line With Horizontal Construction

In the configuration shown in Figure 1, the distance AB is the same as BC, and we can expect the mutual impedance Z_{ab} and Z_{bc} to be equal. However, because of the wider spacing AC, Z_{ac} will not have the same value. The matrix for a 100-mile nontransposed 500 kV line with flat construction is given in (22). The impedance is given in per unit on a 500 kV, 1000 MVA base:

$$\mathbf{Z} = \begin{pmatrix} 0.048 + j0.432 & 0.036 + j0.181 & 0.036 + j0.147 \\ 0.036 + j0.181 & 0.048 + j0.432 & 0.036 + j0.181 \\ 0.036 + j0.147 & 0.036 + j0.181 & 0.048 + j0.432 \end{pmatrix} \quad (22)$$

The symmetrical component matrix in this case is:

$$\mathbf{Z_{syn}} = \mathbf{A\ Z\ A^{-1}} = \begin{pmatrix} 0.12 + j0.771 & 0.01 + j0.06 & 0.01 + j0.06 \\ 0.01 + j0.06 & 0.012 + j0.262 & 0.019 + j0.11 \\ 0.01 + j0.06 & 0.019 + j0.11 & 0.012 + j0.262 \end{pmatrix} \quad (23)$$

In this case, the off-diagonal terms are not zero indicating that there is coupling between the sequence networks for all types of faults. The calculation of a three-phase fault shows the effect of nontransposition on relay reach:

$$\begin{pmatrix} V_a \\ V_b \\ V_c \end{pmatrix} = \begin{pmatrix} 1 \\ a^2 \\ a \end{pmatrix} \qquad \begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} = Z^{-1} \begin{pmatrix} V_a \\ V_b \\ V_c \end{pmatrix} \qquad \begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} = \begin{pmatrix} 0.506 - j3.66 \\ -3.652 + j1.88 \\ 3.317 + j1.725 \end{pmatrix}$$

$$Z1a = \frac{V_a - V_b}{I_a - I_b} \qquad Z1b = \frac{V_b - V_c}{I_b - I_c} \qquad Z1c = \frac{V_c - V_a}{I_c - I_a}$$

$$\left|\frac{Z1a}{Z_{a1}}\right| = 0.952 \qquad \left|\frac{Z1b}{Z_{a1}}\right| = 0.946 \qquad \left|\frac{Z1c}{Z_{a1}}\right| = 1.085$$

The calculation shows that the Phase A and Phase B mho elements underreach by 4.8% and 5.4%, respectively, while the Phase C mho element overreaches by 8.5%. Consequently, the relay settings must take into account the effect of nontransposition.

The three-phase fault calculations using symmetrical components give:

$$\begin{pmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{pmatrix} = Z_{sym}{}^{-1} \begin{pmatrix} V_{a0} \\ V_{a1} \\ V_{a3} \end{pmatrix}$$

$$\begin{pmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{pmatrix} \frac{1}{I_{a1}} = \begin{pmatrix} 0006 + j0.015 \\ 1 \\ 0.046 + j0.072 \end{pmatrix}$$

$$Z_{a1} = \frac{V_{a1}}{I_{a1}} \qquad \frac{\mathrm{Im}(Z_{a1})}{\mathrm{Im}(Z_{sym_{1,1}})} = 0.992$$

Consequently, a negative-sequence current of 8.6% of the three-phase fault current and a 4.7% residual current is produced by a fault on the nontransposed line.

CONCLUSION

The method of symmetrical components depends on line transposition. Relay settings must account for restrictions on sensitivity imposed by nontransposed lines.

REFERENCES

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