Induction Motors: Part I – Analysis

S. E. Zocholl
Schweitzer Engineering Laboratories, Inc.

Revised edition released December 2007

Previous revised edition released May 1997

Original edition released April 1996

INDUCTION MOTORS: PART I - ANALYSIS

by

S. E. Zocholl
Schweitzer Engineering Laboratories, Inc.

INTRODUCTION

Is there ever enough information at hand to analyze a motor problem? Consider this example:

A plant process is shut down for maintenance once every two years. After a scheduled maintenance, an 1800 rpm, 2.4 kV, 1200 hp pump motor tripped on startup. The plant is under new management, and few files are available. However, records list the motor starting time as 20 seconds and the locked rotor time as 14 seconds. The motor bus is fed by a transformer with a 5.6 percent impedance that dips the terminal voltage 75 percent of nominal on starting. The plant manager wants a report and a recommendation to upgrade the protection.

Here is an overview of what needs to be done:

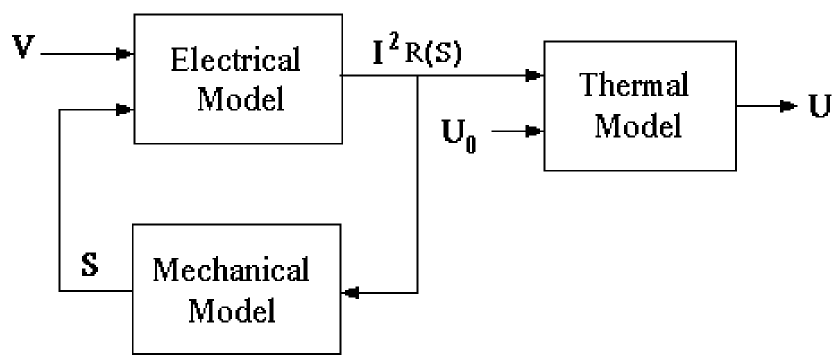

Figure 1: Motor Analysis Block Diagram

Induction motor starting can be analyzed using electrical, mechanical, and thermal models which interact as diagrammed in Figure 1. In the electrical model, the voltage, V, and the slip, S, determine the rotor current. The summation of all torques acting on the motor shaft comprises the mechanical model. Here, the driving torque developed by the motor is resisted by the load torque and the moment of inertia of all the rotating elements, all of which are slip dependent. The thermal model is the equation for heat rise due to current in a conductor determined by the thermal capacity, the thermal resistance, and the slip dependent I^2R watts. As the ultimate protection criteria, the thermal model is used to estimate the rotor temperature, U, resulting from the starting condition with initial temperature U_0. A recursive solution using finite time increments is used because the rotor impedance changes continuously with slip.

As complex as this process may appear, we can add a few standard values and do the complete analysis with the meager information given. In fact, using the SEL-5802 Motor Modeling Program, it can be done in less than a minute. Here is how.

Estimating Input Data

Figure 2 is the menu of the data which defines the electrical and the thermal model of the motor.

To fill in the data, we have used the stated voltage and horse power to calculate the full load current:

$$\mathrm{FLA} = \frac{746 \cdot \mathrm{HP}}{0.8 \cdot \sqrt{3} \cdot \mathrm{V}} = \frac{746 \cdot 1200}{0.8 \cdot \sqrt{3} \cdot 2.400} = 269 \qquad (1)$$

We used 6 times FLA as the locked rotor current and calculated the full load speed using one percent slip at full load. Depending on the class and application of the motor, the locked rotor torque can take on values of 0.8, 1.0, or 1.2. The value of 0.8 is the appropriate value for a pump motor.

MOTOR DATA			
Rated Horse Power	HP	1200	Hp
Rated Speed	FLW	1783	rpm
Synchronous Speed	SynW	1800	rpm
Locked Rotor Torque	LRQ	0.8	pu
Full Load Current	FLA	269	amps
Locked Rotor Current	LRA	1614	amps
Hot Stall Limit Time	To	14	sec.
Cold Stall Limit Time	Ta		sec.
OK			

Figure 2: Menu of Essential Motor Data

Defining the Electrical Model

The program uses the motor data in the motor menu to generate the impedances of the motor including equations for the slip dependent positive- and negative-sequence rotor resistance and reactance:

Locked rotor current: $$I_L = \frac{\mathrm{LRA}}{\mathrm{FLA}} = 6.0 \qquad (2)$$

Rotor resistance at rated speed $$R_0 = \frac{\mathrm{SynW} - \mathrm{FLW}}{\mathrm{SynW}} = 0.01 \qquad (3)$$

Locked rotor resistance $$R_1 = \frac{LRQ}{I_L^2} = 0.022 \quad (4)$$

Stator resistance $$R_S = \frac{R_0}{5} = 0.002 \quad (5)$$

Total series resistance $$R = R_1 + R_S = 0.024 \quad (6)$$

Total series impedance $$Z = \frac{1}{I_L} = 0.167 \quad (7)$$

Total series reactance $$X = \sqrt{Z^2 - R^2} = 0.165 \quad (8)$$

Locked rotor reactance $$X_1 = \frac{X}{2} = 0.0082 \quad (9)$$

Stator reactance $$X_S = X - X_1 = 0.082 \quad (10)$$

Rotor reactance at rated speed $$X_0 = (\tan(12.75°))(1 + R_S) - X_S = 0.145 \quad (11)$$

Positive-sequence rotor resistance $$R_{r+} = (R_1 - R_0) \cdot S + R_0 \quad (12)$$

Positive-sequence rotor reactance $$X_{r+} = (X_1 - X_0) \cdot S + X_0 \quad (13)$$

Negative-sequence rotor resistance $$R_{r-} = (R_1 - R_0) \cdot (2 - S) + R_0 \quad (14)$$

Negative-sequence rotor reactance $$X_{r-} = (X_1 - X_0) \cdot (2 - S) + X_0 \quad (15)$$

The above calculations result in the equivalent circuit shown in Figure 3. We can now have the program calculate the characteristic of rotor torque and current versus slip at rated volts by varying the slip from 1 to 0. This characteristic, shown in Figure 4, will be useful in defining the input watts of the thermal and mechanical models.

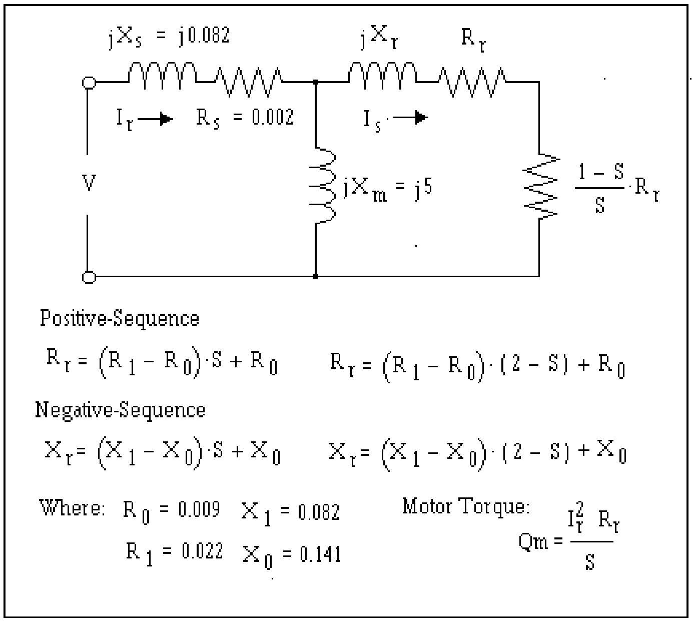

Figure 3: Motor Equivalent Circuit

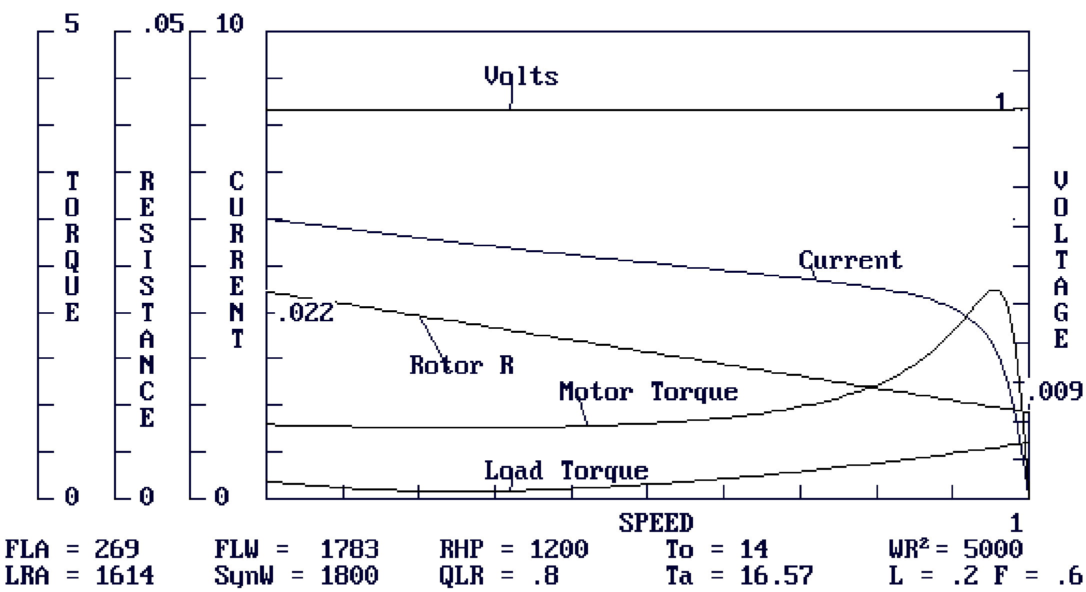

Figure 4: Current and Torque Versus Slip

DEFINING THE MECHANICAL MODEL

The mechanical model is the equation expressing the summation of torques acting on the shaft:

$$(Q_M - Q_L) = M\frac{d\omega}{dt} \tag{16}$$

where Q_M is the motor torque, Q_L is the load torque, M is the combined moment of inertia of the motor and the drive, and ω is the velocity. The equation expressed in time discrete form and solved for slip becomes:

$$(Q_M - Q_L) = M\frac{\omega - \omega_0}{DT} \tag{17}$$

$$\omega = \frac{(Q_M - Q_L)}{M}DT + \omega_0$$
$$S = (1 - \omega) \tag{18}$$

The electromechanical power developed by the rotor is represented by the losses in the variable load resistor in Figure 3. Consequently, the positive-sequence mechanical power is:

$$P_M = I_{r+}^2 \cdot \frac{1-S}{S} R_{r+} \tag{19}$$

where I_{r+}^2 is positive-sequence rotor current. Dividing the power P_M by the velocity, the (1 - S) gives the motor torque:

$$Q_M = \frac{I_{r+}^2 \cdot R_{r+}}{S} \tag{20}$$

Figure 5 shows the typical contour of the load torque versus speed curve of a pump or fan. The load torque is characterized by an initial breakaway torque value, L, and momentary decrease followed by the increase to its final value, F. The program uses the empirical equation:

$$Q_L = L \cdot (1 - \omega)^5 + F \cdot \omega^2 \tag{21}$$

Figure 4 shows the load torque relative to the motor torque. The torque difference (Q_M - Q_L) is the accelerating torque as expressed by equation (16). The accelerating power and the moment of inertia determine the time it takes the motor to reach the peak torque and attain rated speed.

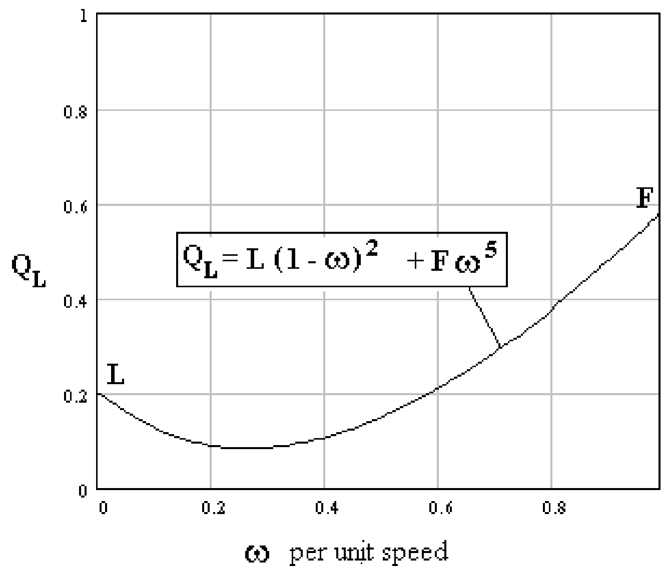

Figure 5: Contour of Load Torque

The mechanical data is entered into the menu shown in Figure 6. Typical values for the load are shown with the moment of the inertia specified in units of lb-ft^2. Since all of the model parameters are specified in per unit of motor base values, the WR^2 is converted to the inertia constant, M, using the following relation:

$$M = \frac{WR^2}{g} \cdot \frac{2\pi}{60} \cdot \frac{SynW}{Q_R} \tag{22}$$

where g is the acceleration due to gravity, SynW is the synchronous speed, and Q_R is the torque calculated from rated speed and horse power:

$$Q_R = 5252 \cdot \frac{RHP}{FLW} \tag{23}$$

With this motor and load, the moment of inertia of 5000 lb-ft^2 produces a 23-second starting time.

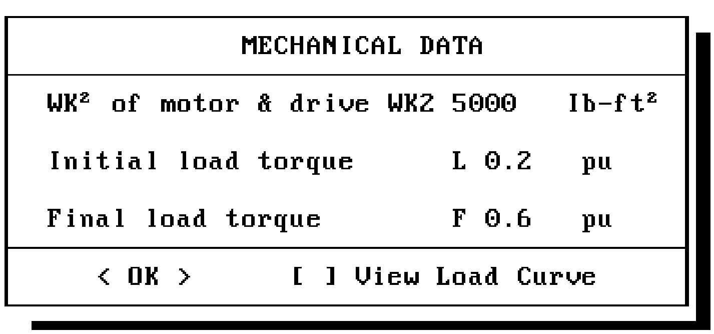

Figure 6: Mechanical Data Menu

DEFINING THE THERMAL MODEL

The differential equation for the temperature rise in a conductor, neglecting heat loss is:

$$I^2 r = C_T \frac{d\theta}{dt} \tag{24}$$

where: $I^2 r$ is the input watts (conductor loss)
C_T is the thermal capacity of the conductor in watt-sec./°C
$d\theta/dt$ is the rate of change of temperature in °C/second

The equation can be integrated to find the temperature rise:

$$\theta = \frac{1}{C_T}\int_0^t I^2 r \cdot dt \tag{25}$$

Therefore, for constant input watts:

$$\theta = \frac{I^2 r}{C_T} t \tag{26}$$

where: θ is the temperature in °C

The heat equation can be represented as a current generator feeding a capacitor. In this analogy, the current is numerically equal to the watts, the capacitor equals the thermal capacitance, and the charge accumulated on the capacitor represents the temperature rise over ambient caused by the watts.

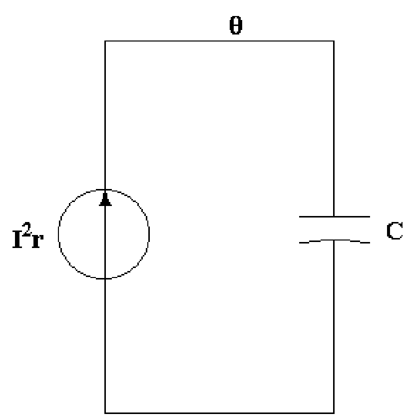

Figure 7: Electrical Analog Circuit

The temperature can be expressed in per unit and be plotted versus per unit current as a time-current characteristic. To do this let:

$$I = m \cdot I_{rated} \tag{27}$$

and substitute for I in equation (24):

$$\theta = \frac{(m \cdot I_{rated})^2 r}{C_T} t \tag{28}$$

Divide equation (28) through by: $\frac{(I_{rated})^2 r}{C_T}$:

$$\theta \frac{C_T}{(I_{rated})^2 r} = m^2 t \tag{29}$$

which can be written simply as:

$$U = m^2 t \tag{30}$$

The above equations show that an I^2t curve can represent a thermal limit. The curve represents a specific temperature expressed in seconds and is a straight line which has a slope of 2 when plotted on log-log paper.

If at locked rotor current M_L, the thermal limit time is T_A, then from (30) the temperature represents the rise over ambient:

$$U_L = M_L^2 T_A \tag{31}$$

If at the locked rotor current M_L, the time to reach the thermal limit is T_O with the conductor initially at operating temperature, then equation (8) becomes:

$$M_L^2 T_A = M_L^2 T_O + U_O \tag{32}$$

and the operating temperature in terms of M_L, T_A, and T_O is:

$$U_O = M_L^2 (T_A - T_O) \tag{33}$$

Equation (30) is the familiar I^2t characteristic which plots as a straight line of slope 2 on log-log paper. The plot shown in Figure 8 represents a specific limiting temperature. The operating temperature, represented by Equation (33), is caused by one per unit current flowing in the thermal resistance of the conductor. Consequently, Equation (33) is the thermal resistance as shown in the electrical analog shown on Figure 9.

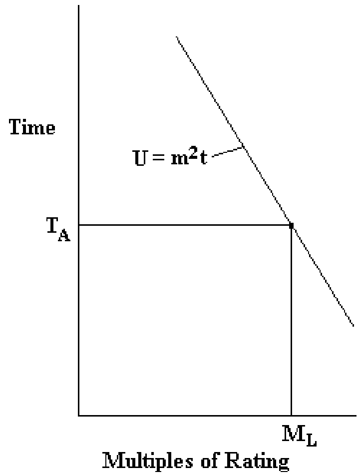

Figure 8: Time-Current Characteristic

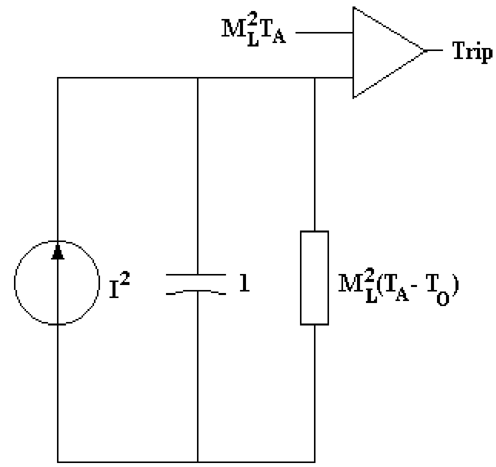

Figure 9: Electrical Analog With Thermal Resistance

Figure 10 shows the thermal model using the slip dependent per unit positive and negative watts input and per unit locked rotor current.

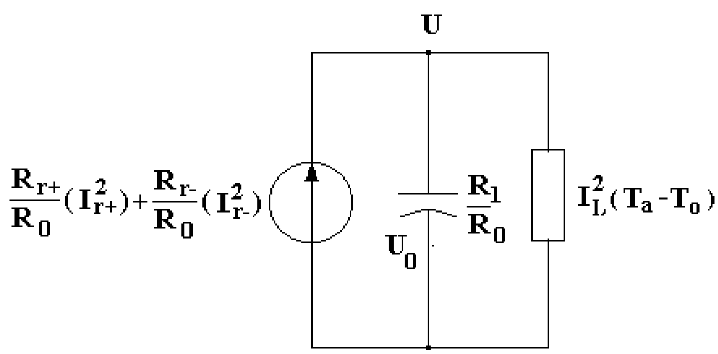

Figure 10: Motor Thermal Model

THE MOTOR ANALYSIS

With the interactive electrical, mechanical, and thermal models defined by the input data, we can now relate the motor current, voltage, and rotor hot spot temperature to time during the start. The plots are calculated by the program and displayed after specifying the parameters in the case menu shown in Figure 11. The plots are shown in Figure 12.

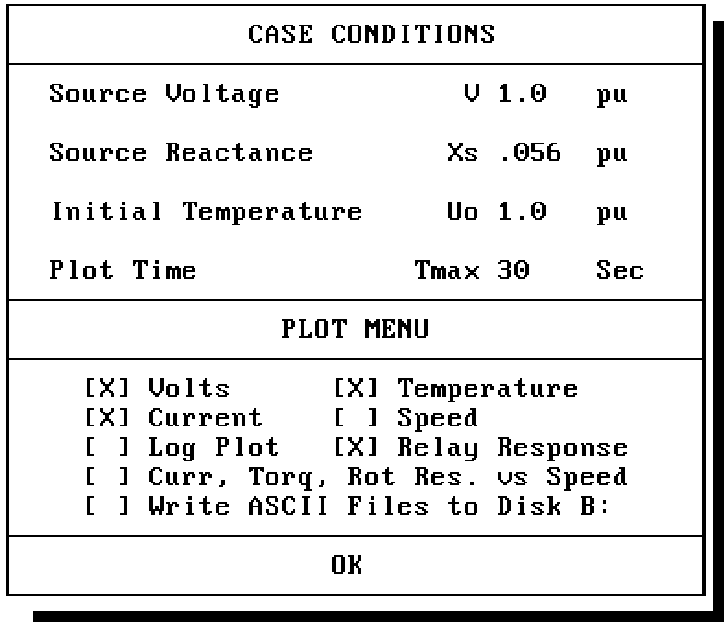

Figure 11: Case Menu

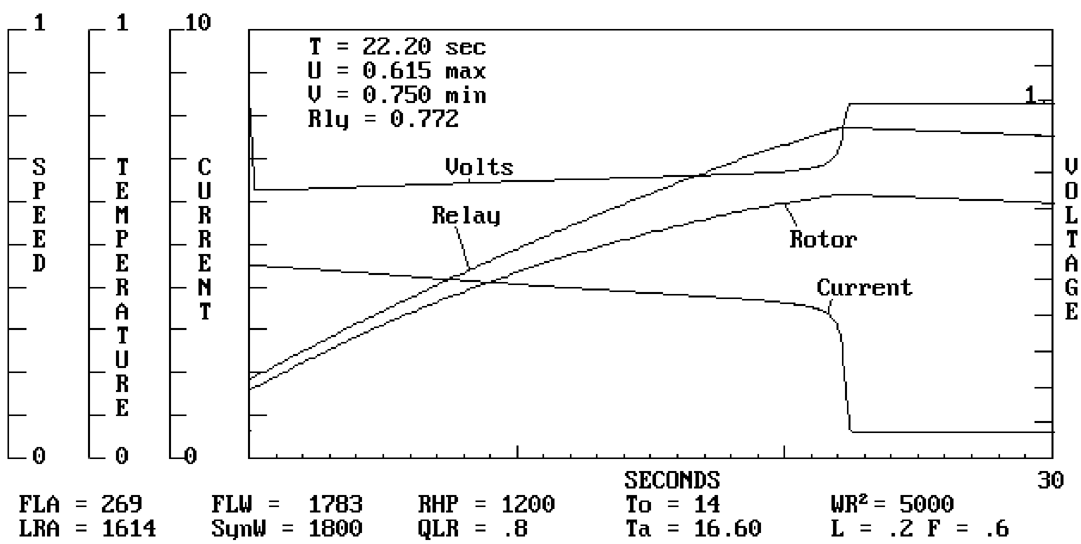

Figure 12: Plot of Motor Voltage, Current, and Rotor Hot Spot Temperature

The plot is essentially an oscillogram of the starting condition. Analyzing the plot we can see that assigned WR^2 gives the approximate starting time and that temperature, U, reaches only 0.615 per unit of the thermal limit. The relay response indicates an adequate relay coordination margin. A program option displays a log-log plot of the SEL-501 characteristics shown in Figure 13 and automatically calculates the settings as shown in Figure 14.

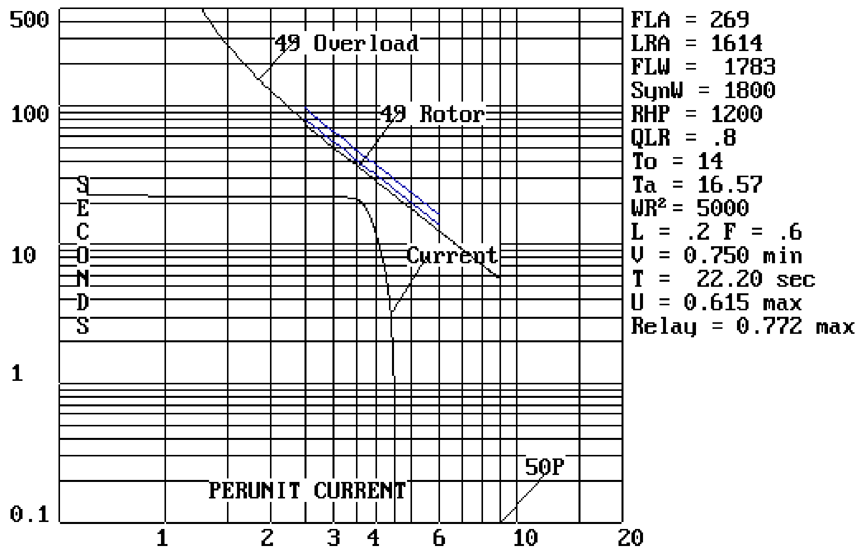

Figure 13: Log Plot of Current and SEL-501 Characteristic

SEL-501 Motor Setting Example

Motor Relay Application	APP	MOT units
CT Ratio	CTR	100
Motor Full Load Amps	FLA	2.6 amps
Motor Locked Rotor Amps	LRA	16 amps
Motor Locked Rotor Time	LRT	14 sec.
Rotor Thermal Time Dial	TD	.9 pu
Motor Service Factor	SF	1.0
Phase Definite-Time OC	50PP	24 amps
Phase Definite-Time Delay	50PD	6 cycles
Phase Instantaneous OC	50H	32 amps
Res. Definite-Time OC	50NP	1.5 amps
Res.Definite-Time Delay	50ND	10 cycles
Residual Instantaneous OC	50NH	5 amps
Optional Negative Sequence Settings:		
Neg. Seq. Definite-Time OC	50QP	1.5 amps
Phase Definite-Time Delay	50QD	240 cycles

Continue

Figure 14: Motor Relay Settings

CONCLUSIONS

The parameters chosen to supplement the motor nameplate data allowed us to verify the known motor voltage and accelerating time. The study determined an adequate relay coordination time when the proper relay thermal protection characteristic is applied. The trace of the starting current verified that the existing moderately inverse relay, set for 14 seconds at locked rotor current, lacked the coordination time for the long duration reduced voltage start.