# Directional Element Design and Evaluation 

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## DIRECTIONAL ELEMENT DESIGN AND EVALUATION

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## Introduction

Directional elements are fundamental to protection-scheme security and selectivity, performing such critical tasks as supervising distance elements and controlling overcurrent elements. Numerical relays, the newest platform for directional elements, form directional characteristics from torque-like quantities and sequence-impedance measurements.

These new relays consolidate several directional elements in one hardware package to determine the direction of all fault types. This consolidation permits the new relays to perform directional calculations and evaluate the results in order to make the best directional decision. Unlike this integrated directional concept, in the electromechanical implementation, a particular directional element has no knowledge of the decisions made by the other directional elements at the same relaying terminal.

This paper presents basic directional element designs. We evaluate various directional element input combinations and analyze how these combinations perform for simple and complex system faults.

We also discuss the following:

- New and traditional directional element input selection
- Classical directional element security problems and remedies
- An improved negative-sequence directional element, which outperforms traditional directional elements in difficult applications
- Positive-sequence superposition directional element performance

Finally, we examine the performance of a new negative-sequence directional element for an actual ground fault on a series-compensated 345 kV transmission line.

## Why Use Directional Elements?

We use directional elements to do the following:

- Determine fault direction
- Supervise distance elements
- Form quadrilateral ground distance characteristics


## Determine Fault Direction

When you apply an overcurrent relay in a looped or networked system, the protective relay needs a directional element to determine fault direction.

Directional overcurrent relays can be set more sensitive than non-directional overcurrent relays. In addition, time-coordination is simplified because the directional element restricts relay response to faults in one direction.

Directional overcurrent elements provide high-speed, primary transmission-line protection with the aid of a communication channel. Directional overcurrent elements, often paired with distance elements, form such schemes as Permissive Overreaching Transfer Trip (POTT) and Directional Comparison Blocking (DCB). These directional overcurrent elements are included in the communication-assisted tripping schemes to overcome the fault resistance limitations of ground distance elements.

## Supervise Distance Elements

Directional elements add security to all distance elements.

## Reverse Ground Faults and Ground Distance Elements

The operating quantities for ground distance elements include residual current $\left(3 \mathrm{I}_{0}\right)$. Long reaching, forward set ground distance elements can pick up for a close-in reverse ground fault on another phase due to the inclusion of $3 \mathrm{I}_{0}$ in the operating quantity. One solution to this problem is to supervise the ground distance elements with an independent directional element.

## Reverse Phase-Phase Faults and Phase Distance Elements

The operating quantities for phase distance elements use phase-phase currents: e.g., the BC-phase distance element uses ( $\mathrm{I}_{\mathrm{B}}-\mathrm{I}_{\mathrm{C}}$ ) in its operating quantity, where $\mathrm{I}_{\mathrm{B}}$ and $\mathrm{I}_{\mathrm{C}}$ are the B -phase and C-phase currents, respectively. For a close-in, reverse CA-phase fault, a forward reaching BC-phase distance element can sense the fault due to the effect of the C-phase current. We can use a negative-sequence directional element to supervise phase distance elements and avoid such misoperations.

## Reverse Three-Phase Faults and Phase Distance Elements

Forward-reaching Mho phase distance elements lack security for reverse three-phase faults if all of the following conditions are fulfilled:

- A significant load current flows into the bus from a weaker source
- The fault includes a small but critical amount of fault resistance
- The polarizing memory voltage expires

Under these conditions, the angle between the polarizing and operating quantities is less than $90^{\circ}$ for forward-reaching phase distance elements. This quantity is the angular difference required to pick up a forward-reaching distance element [1]. One solution to this problem is to supervise the three-phase distance elements with a positive-sequence directional element.

## Form Quadrilateral Ground Distance Characteristics

Quadrilateral ground distance elements are a combination of four elements:

- Directional element (bottom)
- Reactance element (top)
- Right-side resistance element
- Left-side resistance element

The reactance and resistance elements are nondirectional and require a separate directional element to make a directional distance characteristic [1].

## Phase Directional Element Design

Sonnemann [2] describes the popular $90^{\circ}$ connected phase directional element. Table 1 lists the operating and polarizing quantities of these elements.

Table 1 Inputs to the $90^{\circ}$ Connected Phase Directional Element

| Phase | Operating Quantity ( $\mathbf{I}_{\mathbf{O P}}$ ) | Polarizing Quantity ( $\mathbf{V}_{\text {POL }}$ ) |
| :---: | :---: | :---: |
| A | $\mathrm{I}_{\mathrm{A}}$ | $\mathrm{V}_{\text {POLA }}=\mathrm{V}_{\mathrm{BC}}$ |
| B | $\mathrm{I}_{\mathrm{B}}$ | $\mathrm{V}_{\mathrm{POLB}}=\mathrm{V}_{\mathrm{CA}}$ |
| C | $\mathrm{I}_{\mathrm{C}}$ | $\mathrm{V}_{\mathrm{POLC}}=\mathrm{V}_{\mathrm{AB}}$ |

The following equations represent the torque ( $\mathrm{T}_{\text {PHASE }}$ ) calculations for each $90^{\circ}$ connected phase directional element:

$$
\begin{aligned}
& \mathrm{T}_{\mathrm{A}}=\left|\mathrm{V}_{\mathrm{BC}}\right| \cdot\left|\mathrm{I}_{\mathrm{A}}\right| \cdot \cos \left(\angle \mathrm{V}_{\mathrm{BC}}-\angle \mathrm{I}_{\mathrm{A}}\right) \\
& \mathrm{T}_{\mathrm{B}}=\left|\mathrm{V}_{\mathrm{CA}}\right| \cdot\left|\mathrm{I}_{\mathrm{B}}\right| \cdot \cos \left(\angle \mathrm{V}_{\mathrm{CA}}-\angle \mathrm{I}_{\mathrm{B}}\right) \\
& \mathrm{T}_{\mathrm{C}}=\left|\mathrm{V}_{\mathrm{AB}}\right| \cdot\left|\mathrm{I}_{\mathrm{C}}\right| \cdot \cos \left(\angle \mathrm{V}_{\mathrm{AB}}-\angle \mathrm{I}_{\mathrm{C}}\right)
\end{aligned}
$$

where:

$$
\begin{array}{ll}
\mathrm{I}_{\mathrm{A}}, \mathrm{I}_{\mathrm{B}}, \mathrm{I}_{\mathrm{C}} & =\text { A-, } \mathrm{B}-\text {, and } \mathrm{C}-\text { phase currents, respectively. } \\
\mathrm{V}_{A}, V_{B}, V_{C} & =A-, \text { B-, and } C \text {-phase voltages, respectively. } \\
V_{A B}, V_{B C}, V_{C A} & =\text { voltage differences } V_{A}-V_{B}, V_{B}-V_{C}, \text { and } V_{C}-V_{A}, \text { respectively. }
\end{array}
$$

Each directional element declares a forward fault condition if the torque sign is positive and a reverse fault condition if the torque sign is negative.

Now let us examine the response of the $90^{\circ}$ connected phase directional relays for a forward three-phase fault on the system shown in Figure 1. In this example, the three-phase fault is located at $\mathrm{m}=0.3$ where m is the per-unit line distance to the fault.


Relay 1

$$
\begin{array}{ll}
\mathrm{I}_{\mathrm{OPA}}=33.5 \mathrm{~A} \angle-90^{\circ} & \mathrm{V}_{\mathrm{POLA}}=69.6 \mathrm{~V} \angle-90^{\circ} \\
\mathrm{I}_{\mathrm{OPB}}=33.5 \mathrm{~A} \angle+150^{\circ} & \mathrm{V}_{\mathrm{POLB}}=69.6 \mathrm{~V} \angle+150^{\circ} \\
\mathrm{I}_{\mathrm{OPC}}=33.5 \mathrm{~A} \angle+30^{\circ} & \mathrm{V}_{\mathrm{POLC}}=69.6 \mathrm{~V} \angle+30^{\circ}
\end{array}
$$

Figure 1 Relay Quantities for a Forward Direction Three-Phase Fault at $\mathrm{m}=0.3$
For balanced faults, all three $90^{\circ}$ connected phase directional elements agree on the fault direction. This direction agreement is always valid for balanced faults.

Do these $90^{\circ}$ connected phase directional elements agree if the fault is other than a balanced fault?

Warrington [3] identifies a system-dependent fault condition that produces a misoperation for $90^{\circ}$ connected phase directional elements [Figure 2(a)]. This condition is a reverse single-line-ground (SLG) fault where the remote infeed current from Bus S is predominantly zero-sequence.

For the fault shown in Figure 2(a), the phase directional declaration must be forward for Bus S elements and reverse for Bus R elements.

In the phasor diagrams in Figure 2(b), Phase A of Relay 1, and Phases B and C of Relay 2 declare the fault direction as forward. Phase B and C of Relay 1, and Phase A of Relay 2 declare the fault direction as reverse. Notice that the collection of phase directional elements at each terminal do not agree on the fault direction.

This disagreement is due to the large angular separation of the polarizing quantities. With these polarizing and operating quantities, at least one directional element at each terminal makes an incorrect directional decision.


Figure 2(a) Reverse SLG Fault with a Strong Remote Zero-Sequence Infeed


Figure 2(b) Reverse SLG Fault Produces Misoperation for $90^{\circ}$ Connection Phase Directional Elements of Figure 2(a)

Incorrect Phase B and C directional decisions at Bus $R$ can cause an undesired trip of the unfaulted line if the fault current is above the phase overcurrent pickup threshold. In this example, the fault current is below the pickup of a typical phase overcurrent element. However, shortening the line length would increase the fault current enough to operate a phase overcurrent element.

## A Better Phase Directional Element

For the three-phase fault discussed earlier, all $90^{\circ}$ connected phase directional element declarations agreed, yet for the out-of-section SLG fault they did not. How can we prevent phase directional element disagreements from sacrificing scheme security?

The following are three possible solutions to this problem:
Solution 1 Require agreement of all phase directional elements before declaring phase fault direction.

Solution 1 offers the desired security for the SLG fault case but would block the phase directional elements from operating for phase-phase faults. It requires a separate directional element to discriminate phase-phase fault direction.

Solution 2 Supervise each phase directional decision with a phase-phase overcurrent element (in the SLG case, the phase-phase current is zero).

While Solution 2 is feasible in a numerical relay, it is not practical in an electromechanical relay.

Solution 3 Block three-phase directional decisions by the phase directional elements if the zero-sequence current is above some threshold.

Solution 3 is satisfactory if the phase relays are not required to operate for phase-phase-ground faults.

Now let us consider a fourth solution that builds on the first solution but reduces the total number of phase directional elements.

Positive-sequence voltages and currents are the only sequence-quantities present for balanced faults. We can use these quantities to produce a single three-phase directional element and replace the three separate $90^{\circ}$ connected phase directional elements. Using a balanced fault directional element to supervise phase overcurrent and distance elements does require a separate directional element for unbalanced faults. However, the total number of phase directional elements is reduced by one.

Table 2 shows the inputs to a positive-sequence directional element.
Table 2 Inputs to a Positive-Sequence Directional Element

| Phase | Operating Quantity $\left(\mathbf{I}_{\mathbf{1 O P}}\right)$ | Polarizing Quantity ( $\mathbf{V}_{\mathbf{1 P O L}}$ ) |
| :---: | :---: | :---: |
| Three-Phase | $3 \mathrm{I}_{1} \cdot\left(1 \angle \mathrm{Z}_{\mathrm{LI}}\right)$ | $3 \mathrm{~V}_{1}$ |

Equation (1) represents the calculated positive-sequence directional element torque (T32P).

$$
\begin{equation*}
\mathrm{T} 32 \mathrm{P}=\left|3 \mathrm{~V}_{1}\right| \cdot\left|3 \mathrm{I}_{1}\right| \cdot \cos \left[\angle 3 \mathrm{~V}_{1}-\left(\angle 3 \mathrm{I}_{1}+\angle \mathrm{Z}_{\mathrm{L} 1}\right)\right] \tag{1}
\end{equation*}
$$

where:

$$
\begin{array}{ll}
3 \mathrm{I}_{1} & =\text { Positive-sequence current: } 3 \mathrm{I}_{1}=\left(\mathrm{I}_{\mathrm{A}}+\mathrm{a} \cdot \mathrm{I}_{\mathrm{B}}+\mathrm{a}^{2} \cdot \mathrm{I}_{\mathrm{C}}\right) . \\
3 \mathrm{~V}_{1} & =\text { Positive-sequence voltage: } 3 \mathrm{~V}_{1}=\left(\mathrm{V}_{\mathrm{A}}+\mathrm{a} \cdot \mathrm{~V}_{\mathrm{B}}+\mathrm{a}^{2} \cdot \mathrm{~V}_{\mathrm{C}}\right) . \\
\mathrm{a} & =1 \angle 120^{\circ} . \\
\angle \mathrm{Z}_{\mathrm{L} 1} & =\text { Positive-sequence line angle. }
\end{array}
$$

The sign of T32P is positive for forward three-phase faults and negative for reverse three-phase faults. As an additional security step, the magnitude of T32P should exceed a minimum threshold before the element considers the directional decision valid. This requirement avoids erroneous directional decisions when either $\left|\mathrm{V}_{1 \mathrm{POL}}\right|$ or $\left|\mathrm{I}_{1 \mathrm{OP}}\right|$ is so small as to make their angles unreliable.

## T32P Response to the SLG Fault Shown in Figure 2(A)

How would the positive-sequence directional element at Bus R react to the same reverse SLG fault, presented earlier? Recall that the magnitude of T32P must exceed a minimum torque threshold. Because $\mathrm{I}_{10 \mathrm{P}}$ is zero for this reverse SLG fault, the positive-sequence directional element would not give an output.

## T32P Response to Zero-Voltage Three-Phase Faults

If the polarizing quantity of the positive-sequence directional element does not have memory, T32P cannot operate for a three-phase fault where the magnitudes of all three phase voltages are almost zero (neither could the $90^{\circ}$ connected phase directional elements). Using positivesequence memory voltage ( $\mathrm{V}_{\text {1MEM }}$ ) for $\mathrm{V}_{1 \text { PoL }}$ overcomes a failure to operate for a close-in threephase fault. Simply substitute $\mathrm{V}_{\text {1мем }}$ for $3 \mathrm{~V}_{1}$ in (1), and the T32P element can operate correctly for a close-in three-phase fault until the memory expires.

## Out-of-Section Phase-Phase Fault with Heavy Load Affects T32P Adversely

The T32P element may need some additional supervision if the fault does not involve all three phases and the fault current magnitudes approach that of load.

Figure 3 shows the T32P element performance for a BC fault (Source Impedances: $\mathrm{Z}_{\mathrm{S} 1}=\mathrm{Z}_{\mathrm{R} 1}=$ $0.8 \angle 90^{\circ} \Omega$ and Line Impedances: $\mathrm{Z}_{\mathrm{L} 1}=\mathrm{Z}_{\mathrm{L} 2}=4 \angle 90^{\circ} \Omega$ ). For this example, load is flowing from left to right $\left(\delta=30^{\circ}\right)$, and the prefault load magnitude is 4.8 A secondary. The desired fault direction declarations are forward for Relay 1 and reverse for Relay 2.

The T32P element for Relay 2 declares the fault forward ( $\mathrm{T} 32 \mathrm{P}=+148.5$ ). The element misoperates because $\Theta$, the angle between $V_{1}$ and $\left(\mathrm{I}_{1} \cdot 1 \angle \mathrm{Z}_{\mathrm{L}}\right)$, is less than $90^{\circ}$ (Figure 3). This incorrect directional decision is due to load and fault current magnitudes being approximately equal in Line 1. Moving the BC fault on Line 2 closer to Bus R allows T32P for Relay 2 to correctly identify the fault as reverse.


|  | Relay 1 at Bus S |  | Relay 2 at Bus R |  |
| :--- | :--- | :--- | :--- | :--- |
|  |  |  | $57.0 \mathrm{~V} \angle+25^{\circ}$ |  |
| Positive - Sequence $\mathrm{V}: \mathrm{V}_{1}$ |  | $55.0 \mathrm{~V} \angle+5^{\circ}$ |  |  |
| Positive - Sequence $\mathrm{I}: \mathrm{I}_{1}$ |  | $4.8 \mathrm{~A} \angle+9^{\circ}$ |  | $4.8 \mathrm{~A} \angle-171^{\circ}$ |
| Negative - Sequence $\mathrm{V}: \mathrm{V}_{2}$ | $8.4 \mathrm{~V} \angle+13^{\circ}$ |  | $10.5 \mathrm{~V} \angle+13^{\circ}$ |  |
| Negative - Sequence $\mathrm{I}: \mathrm{I}_{2}$ |  | $0.5 \mathrm{~A} \angle+103^{\circ}$ |  | $0.5 \mathrm{~A} \angle-77^{\circ}$ |




Figure 3 Out-of-Section BC Fault Adversely Affects T32P Element for Relay 2

## How Can We Avoid This Incorrect Directional Declaration By T32P at Relay 2?

## Phase Directional Element for Unbalanced Faults

We need a separate directional element to determine unbalanced fault direction. The decision of this element must overrule the T32P element for unbalanced faults.

What quantities should we use as inputs to an unbalanced fault directional element? Determine what quantities are present for phase-phase and phase-phase-ground faults (see Table 3).

Table 3 Available Sequence Component Quantities Unbalance Phase Faults

| Sequence Quantity | Phase-Phase Faults | Phase-Phase-Ground Faults |
| :---: | :---: | :---: |
| $\mathrm{V}_{1}$ | Yes | Yes |
| $\mathrm{V}_{2}$ | Yes | Yes |
| $\mathrm{V}_{0}$ | No | Yes |
| $\mathrm{I}_{1}$ | Yes | Yes |
| $\mathrm{I}_{2}$ | Yes | Yes |
| $\mathrm{I}_{0}$ | No | Yes |

From Table 3, we see that only positive- and negative-sequence quantities are available for both unbalanced phase fault types. As positive-sequence load quantities mislead a directional element for unbalanced faults, only negative-sequence quantities remain as viable inputs to an unbalanced fault directional element.

Table 4 shows the inputs to a traditional negative-sequence directional element, and (2) shows the torque expression.

Table 4 Inputs to a Traditional Negative-Sequence Directional Element

| Operating Quantity $\left(\mathbf{I}_{\mathbf{O P}}\right)$ | Polarizing Quantity ( $\mathbf{V}_{\text {POL }}$ ) |
| :---: | :---: |
| $3 \mathrm{I}_{2} \cdot\left(1 \angle \mathrm{Z}_{\mathrm{L} 1}\right)$ | $-3 \mathrm{~V}_{2}$ |
| $\mathrm{~T} 32 \mathrm{Q}=\left\|3 \mathrm{~V}_{2}\right\| \cdot\left\|3 \mathrm{I}_{2}\right\| \cdot \cos \left[\angle-3 \mathrm{~V}_{2}-\left(\angle 3 \mathrm{I}_{2}+\angle \mathrm{Z}_{\mathrm{LI}}\right)\right]$ |  |

where:

$$
\begin{aligned}
& 3 \mathrm{I}_{2}=\text { Negative-sequence current: } 3 \mathrm{I}_{2}=\left(\mathrm{I}_{\mathrm{A}}+\mathrm{a}^{2} \cdot \mathrm{I}_{\mathrm{B}}+\mathrm{a} \cdot \mathrm{I}_{\mathrm{C}}\right) . \\
& -3 \mathrm{~V}_{2}=\text { Negative-sequence voltage: }-3 \mathrm{~V}_{2}=\left(\mathrm{V}_{\mathrm{A}}+\mathrm{a}^{2}+\mathrm{V}_{\mathrm{B}}+\mathrm{a} \cdot \mathrm{~V}_{\mathrm{C}}\right) \cdot\left(1 \angle 180^{\circ}\right) .
\end{aligned}
$$

T32Q is positive for forward faults and negative for reverse faults. As with the T32P element, the magnitude of T32Q must exceed a minimum threshold. Using $V_{2}$ and $I_{2}$ from Figure 3 in (2), we calculate $\mathrm{T} 32 \mathrm{Q}=-50.13$ for Relay 2 . The negative sign of T32Q correctly indicates a reverse fault.

Note: The $3 \mathrm{~V}_{2}$ quantity used in (2) is multiplied by $1 \angle 180^{\circ}$ to make the torque calculation positive for forward faults.

Combining the positive- and negative-sequence directional element torques creates a net- phase directional torque. This combined phase directional torque, T32P and T32Q, replaces the individual phase directional elements. Call this net-phase torque result T32PQ. However, simply adding the positive- and negative-sequence directional element torques does not adequately restrain T32P from misoperating for the out-of-section phase-phase fault. We must weight the T32Q torque greater than the T32P torque to gain the required security.

We can divide the positive-sequence torque by some number greater than one and add this weighted T32P torque to the calculated T32Q torque. For example, we can divide T32P by four. Using the net-phase torque concept for Relay 2 at Bus R, we calculate the following result:

$$
\mathrm{T} 32 \mathrm{PQ}=\mathrm{T} 32 \mathrm{Q}+\mathrm{T} 32 \mathrm{P} / 4=-50.13+37.13=-13.01
$$

Because T32PQ $<0$, Relay 2 declares a reverse fault condition.
We have shown the negative-sequence directional element being combined with the positivesequence directional element to form a secure net-phase torque result. If the protective relay does not use a positive-sequence directional element, the T32Q element could still be used as a separate unbalanced directional element for unbalanced fault protection.

## Alternate Representation of the Negative-Sequence Directional Element

Equation (2) uses a cosine term to represent the traditional negative-sequence directional element torque. The cosine term conveniently defines the zero- and maximum-torque boundaries: given two inputs, $A$ and $B$, separated by an angle $\Theta, \cos \Theta$ is one (positive maximum) when $A$ and $B$ are in phase and zero when $A$ and $B$ are in quadrature.
$\operatorname{Re}\left(A \cdot B^{*}\right)$ is an equivalent representation of this cosine term. Both of them represent the torque produced by an induction cup element with inputs A and B . In a numerical relay implementation, the execution time for the $\operatorname{Re}\left(A \cdot B^{*}\right)$ calculation is far faster than that of the cosine term (in fact, the cosine term may not be directly available in all microprocessors). The following discussion builds on the idea of developing a negative-sequence directional element in a numerical relay.

Given phasors A 2 and B 2 , consider the following complex negative-sequence torque product, [DIGT2]:

$$
\begin{aligned}
\text { DIGT2 } & =\mathrm{A} 2 \cdot \mathrm{~B} 2^{*} \\
& =\left(\mathrm{A} 2_{\mathrm{X}}+\mathrm{j} \cdot \mathrm{~A} 2_{\mathrm{Y}}\right) \cdot\left(\mathrm{B} 2_{\mathrm{X}}-\mathrm{j} \cdot \mathrm{~B} 2_{\mathrm{Y}}\right) \\
& =\mathrm{A} 2_{\mathrm{X}} \cdot \mathrm{~B} 2_{\mathrm{X}}+\mathrm{A} 2_{\mathrm{Y}} \cdot \mathrm{~B} 2_{\mathrm{Y}}+\mathrm{j} \cdot\left(\mathrm{~A} 2_{\mathrm{Y}} \cdot \mathrm{~B} 2_{\mathrm{X}}-\mathrm{A} 2_{\mathrm{X}} \cdot \mathrm{~B} 2_{\mathrm{Y}}\right)
\end{aligned}
$$

where:

* = complex conjugate (changes the angle sign).
$\mathrm{B} 2=\mathrm{I}_{2} \cdot\left(1 \angle \mathrm{Z}_{\mathrm{L} 1}\right)$.
A2 $=V_{2}$
The angle of the product (A2•B2*) is the angle of A2/B2 and the angle by which A2 leads B2.
Without loss of generality, assume that our phase reference is B2, and that phasor A2 leads phasor $B$ by angle $\Theta$. In this frame of reference,
$\mathrm{B} 2_{\mathrm{x}}=|\mathrm{B} 2|$
$B 2{ }_{\mathrm{Y}}=$
0
$\mathrm{A} 2_{\mathrm{X}}=|\mathrm{A} 2| \cdot \cos \Theta$
$\mathrm{A} 2_{\mathrm{Y}}=\quad|\mathrm{A} 2| \cdot \sin \Theta$
and
DIGT2 $=\mid$ A2 $|\cdot|$ B2 $|\cdot \cos \Theta+j \cdot|$ A2 $|\cdot|$ B2 $\mid \cdot \sin \Theta$
Separate the real and imaginary parts of DIGT2 $=\mathrm{P} 2+\mathrm{j} \cdot \mathrm{Q} 2$ :

$$
\begin{aligned}
& \mathrm{P} 2=|\mathrm{A}| \cdot|\mathrm{B}| \cdot \cos \Theta=\operatorname{Re}(\mathrm{A} 2 \cdot \mathrm{~B} 2 *) \\
& \mathrm{Q} 2=|\mathrm{A}| \cdot|\mathrm{B}| \cdot \sin \Theta=\operatorname{Im}\left(\mathrm{A} 2 \cdot \mathrm{~B} 2^{*}\right)
\end{aligned}
$$

Both P2 and Q2 are two-input phase angle comparators. The P-comparator has a maximum torque when A 2 and B 2 are in phase, and zero torque when A 2 and B 2 are in quadrature.

Consider a BC-phase fault at $\mathrm{m}=0$ on the $90^{\circ}$ radial power system in Figure 4. As measured by Relay $1, \mathrm{~V}_{2}$ lags $\mathrm{I}_{2}$ by the source angle $\left(\mathrm{Z}_{\mathrm{S} 2}=0.8 \angle 90^{\circ} \Omega\right)$. After adjusting $\mathrm{I}_{2}$ by the maximum torque angle (source angle) and changing the sign (using the complex conjugate term), A2 and B2* are $180^{\circ}$ out-of-phase. Of the two comparators, P2 and Q2, only P2 has a non-zero value, showing us that we want to use the P2 comparator and that forward faults are indicated by the negative sign of DIGT2.


Figure 4 Negative-Sequence Quantities Forward BC Fault
This fault-direction-sign convention is opposite of the directional elements discussed earlier. If we use $-\mathrm{V}_{2}$ instead of $\mathrm{V}_{2}$, the sign convention for P 2 is the same as the traditional negativesequence directional element.

## Ground Directional Element Design

Ground directional elements supervise sensitive residual overcurrent and ground distance elements. The pickup threshold of a ground overcurrent element is typically set low to detect high-resistance faults. Because the ground protection element is sensitive, the directional element must be secure.

## Ground Directional Element Input Choices

Which two quantities should we use as inputs to a ground directional element? Similar to the unbalanced fault directional element discussion, we need to examine what quantities are present for faults involving ground. The only two fault types we need to consider are single-line-ground and phase-phase-ground faults. Table 5 lists the available sequence quantities for these faults.

Table 5 Available Sequence Quantities for Faults Involving Ground

| Quantity | Phase-Phase-Ground Faults | Single-Line-Ground Faults |
| :---: | :---: | :---: |
| $\mathrm{V}_{1}$ | Yes | Yes |
| $\mathrm{V}_{2}$ | Yes | Yes |
| $\mathrm{V}_{0}$ | Yes | Yes |
| $\mathrm{I}_{1}$ | Yes | Yes |
| $\mathrm{I}_{2}$ | Yes | Yes |
| $\mathrm{I}_{0}$ | Yes | Yes |

## Sequence-Quantities as Inputs to Ground Directional Elements

All sequence-quantities are available for faults involving ground. The positive-sequence quantities are adversely affected by load and should be avoided, leaving only zero- and negativesequence quantities as possible inputs to a ground directional element.

## Zero-Sequence Voltage Polarized Ground Directional Element

A zero-sequence voltage polarized ground directional element uses $\mathrm{V}_{0}$ or $3 \mathrm{~V}_{0}$ as the polarizing reference. Equation (3) represents the torque for a zero-sequence voltage polarized directional element:

$$
\begin{equation*}
\mathrm{T} 32 \mathrm{~V}=\left|3 \mathrm{~V}_{0}\right| \cdot\left|3 \mathrm{I}_{0}\right| \cdot \cos \left[\angle-3 \mathrm{~V}_{0}-\left(\angle 3 \mathrm{I}_{0}+\angle \mathrm{Z}_{\mathrm{L} 0}\right)\right] \tag{3}
\end{equation*}
$$

where:

$$
\begin{aligned}
& 3 \mathrm{~V}_{0}=\text { Zero-sequence voltage: } 3 \mathrm{~V}_{0}=\left(\mathrm{V}_{\mathrm{A}}+\mathrm{V}_{\mathrm{B}}+\mathrm{V}_{\mathrm{C}}\right) . \\
& 3 \mathrm{I}_{0}=\text { Zero-sequence current: } 3 \mathrm{I}_{0}=\left(\mathrm{I}_{\mathrm{A}}+\mathrm{I}_{\mathrm{B}}+\mathrm{I}_{\mathrm{C}}\right) . \\
& \angle \mathrm{Z}_{\mathrm{L} 0}=\text { Zero-sequence line angle. }
\end{aligned}
$$

The sign of T32V is positive for forward faults and negative for reverse faults.
If the polarizing voltage magnitude becomes too small, its angle becomes unreliable. Remote ground faults present the ground directional relay with the lowest polarizing voltage magnitude.

The traditional solution to low polarizing voltage magnitude applications has been to use a current polarized directional element. We present an alternative relay algorithm solution to this low magnitude polarizing voltage condition later in this paper.

## Zero-Sequence Current Polarized Ground Directional Element

A zero-sequence current polarized ground directional element measures the phase angle difference between the line residual current $\left(3 \mathrm{I}_{0}\right)$ and an external polarizing source current $\left(\mathrm{I}_{\mathrm{POL}}\right)$.

The zero-sequence current polarized ground directional element differs from the negative- and zero-sequence voltage polarized directional elements in that $3 \mathrm{I}_{0}$ does not require phase shifting by the line angle. Equation (4) represents the torque equation for a zero-sequence current polarized directional element:

$$
\begin{equation*}
\mathrm{T} 32 \mathrm{I}=\left|\mathrm{I}_{\mathrm{POL}}\right| \cdot\left|3 \mathrm{I}_{0}\right| \cdot \cos \left(\angle \mathrm{I}_{\mathrm{POL}}-\angle 3 \mathrm{I}_{0}\right) \tag{4}
\end{equation*}
$$

T32I is positive for forward faults and negative for reverse faults.

## Dual Polarized Zero-Sequence Directional Element

A dual polarized zero-sequence directional element is the combination of a zero-sequence voltage polarized directional element and a zero-sequence current polarized directional element. This element provides more flexibility than a single method of zero-sequence polarization.

For example, if the zero-sequence voltage magnitude presented to the relay for a remote fault is too low, the torque produced by the zero-sequence voltage directional element may be too low to cross its minimum torque threshold. If you are using a dual polarized relay in this situation, you rely solely on the zero-sequence current polarized directional element. If the source for zerosequence polarizing current is switched out-of-service while the transmission line remains inservice, you must rely on the zero-sequence voltage polarized directional element. In the situation where the current polarizing source is switched out-of-service and a remote ground fault still does not produce enough zero-sequence polarizing voltage, you must consider an alternative polarizing technique.

## Zero-Sequence Polarized Directional Elements in Parallel Line Applications

Parallel lines with common buses at both line ends do not cause zero-sequence polarized directional elements to make incorrect fault direction declarations. Zero-sequence polarized directional elements are reliable in these applications (assuming the minimum torque threshold is crossed for all faults). Figure 5 shows the direction of zero-sequence current flow for an SLG fault on Line 1. The direction of zero-sequence current flow in both sources is always the same direction for SLG faults anywhere on Lines 1 or 2 .


Figure 5 Zero-Sequence Polarization is Reliable in Parallel Line Applications with Common Busses at Both Transmission Line Ends

Zero-sequence polarized directional elements can misoperate in parallel lines with high zerosequence mutual coupling $\left(\mathrm{Z}_{\mathrm{OM}}\right)$ and isolated zero-sequence sources. Figure 6 shows parallel lines with isolated zero-sequence sources and an SLG fault on Line 1.


Figure 6 Zero-Sequence Polarization Is not Reliable in Parallel Line Applications With Isolated Zero-Sequence Sources

From Figure 6, notice that the zero-sequence current in each source is in phase with the zerosequence current of the corresponding relay: the zero-sequence directional elements at all terminals declare a forward fault condition. Undesired trips may occur in the unfaulted parallel line [4].

Figure 7 illustrates an isolated zero-sequence source condition. Zero-sequence source isolation also occurs in parallel lines with a single common bus after the breaker closest to the common bus opens.


Figure 7 Zero-Sequence Source Isolation in Parallel Lines with One Common Bus

## Negative-Sequence Directional Element for Ground Faults

The negative-sequence directional element, described earlier as an unbalanced phase fault directional element, is also effective as a ground directional element.

## Why Use Negative-Sequence Quantities for Ground Directional Polarization?

Negative-sequence polarized directional elements have the following two major advantages when compared to zero-sequence voltage polarized directional elements:

1. Negative-sequence directional elements are insensitive to zero-sequence mutual coupling associated with parallel transmission line applications. They are suitable for isolated zerosequence source systems.
2. If the bus behind the relay location is a strong zero-sequence source, the negative-sequence voltage available at the relay location is higher than the zero-sequence voltage.

## Compensating for Low Polarizing Voltage Magnitude

The traditional negative-sequence directional element (32Q) works well in most applications. However, when the negative-sequence source behind the relay terminal is strong (low impedance), the amount of negative-sequence voltage measured by the relay for a remote fault can be too low to properly polarize a traditional negative-sequence directional element. You can make a similar observation for zero-sequence voltage polarized directional relays.

This reduction of the polarizing voltage is most pronounced for remote faults. Figure 8 shows the negative-sequence voltage profile for an SLG fault near the remote terminal.


Figure 8 Negative-Sequence Voltage Profile for SLG Fault at the Remote Terminal
While the magnitude of $\mathrm{V}_{2}$ at Relay 1 location may be small for the remote fault shown in Figure 8 , the negative-sequence current magnitude is large. If the relay could be "moved" towards the fault, the magnitude of $V_{2}$ available to the relay would increase. Since we cannot physically move the relay, we need to think of a different approach.

To overcome low $\mathrm{V}_{2}$ magnitude, one type of directional relay adds a compensating quantity that boosts $\mathrm{V}_{2}$ by $-\left(\alpha \cdot \mathrm{Z}_{\mathrm{L} 1} \cdot \mathrm{I}_{2}\right)$. The amount of compensation (or $\mathrm{V}_{2}$ boost) is controlled by the $\alpha$ constant. Including this compensating quantity in the negative-sequence directional element creates a compensated negative-sequence directional element.

Equation (5) shows the torque for a compensated negative-sequence directional element.

$$
\begin{equation*}
\mathrm{T} 32 \mathrm{QC}=\operatorname{Re}\left[\left(\mathrm{V}_{2}-\alpha \cdot \mathrm{Z}_{\mathrm{L} 1} \cdot \mathrm{I}_{2}\right) \cdot\left(\mathrm{Z}_{\mathrm{L} 1} \cdot \mathrm{I}_{2}\right)^{*}\right] \tag{5}
\end{equation*}
$$

For forward faults, $-\left(\alpha \cdot \mathrm{Z}_{\mathrm{L} 1} \cdot \mathrm{I}_{2}\right)$ adds with $\mathrm{V}_{2}$ to increase the negative-sequence voltage used in the directional calculation. For reverse faults, $-\left(\alpha \cdot \mathrm{Z}_{\mathrm{L} 1} \cdot \mathrm{I}_{2}\right)$ subtracts from the measured $\mathrm{V}_{2}$. You cannot set $\alpha$ too large for forward faults. However, you can set $\alpha$ so large as to make a reverse fault appear as a forward fault. This incorrect directional decision is a result of $-\left(\alpha \cdot \mathrm{Z}_{\mathrm{L} 1} \cdot \mathrm{I}_{2}\right)$ having a greater magnitude and opposite sign than the measured $\mathrm{V}_{2}$ for reverse faults. Therefore, when applying a compensated directional element, you must pay close attention to the $\alpha$ setting to ensure directionality.

How large can you safely set the $\alpha$ factor? Calculate the maximum $\alpha$ setting considering a closein reverse fault on the two-terminal system shown at the top of Figure 9. The sequence network for this fault is also shown in Figure 9. The negative-sequence current $\left(I_{2}\right)$ measured by the relay for this fault is $-\mathrm{I}_{\mathrm{R} 2}$.


Figure 9 Single-Line Diagram and Sequence Network for Reverse SLG Fault
Calculate the maximum $\alpha$ setting or where $\left(\mathrm{V}_{2}-\alpha \cdot \mathrm{Z}_{\mathrm{L} 1} \cdot \mathrm{I}_{2}\right)=0$.

$$
\begin{aligned}
{\left[\mathrm{V}_{2}-\alpha \cdot \mathrm{Z}_{\mathrm{L} 1} \cdot\left(-\mathrm{I}_{\mathrm{R} 2}\right)\right] } & =0 \\
\mathrm{~V}_{2} & =\alpha \cdot \mathrm{Z}_{\mathrm{L} 1} \cdot\left(-\mathrm{I}_{\mathrm{R} 2}\right)
\end{aligned}
$$

For this fault, $\mathrm{V}_{2}$ measured is equal to $\left[\left(-\mathrm{I}_{\mathrm{R} 2}\right) \cdot\left(\mathrm{Z}_{\mathrm{R} 2}+\mathrm{Z}_{\mathrm{L} 2}\right)\right]$. Substituting,

$$
\begin{aligned}
\mathrm{I}_{\mathrm{R} 2} \cdot\left(\mathrm{Z}_{\mathrm{R} 2}+\mathrm{Z}_{\mathrm{L} 2}\right) & =\alpha \cdot \mathrm{Z}_{\mathrm{L} 2} \cdot \mathrm{I}_{\mathrm{R} 2} \\
\left(\mathrm{Z}_{\mathrm{R} 2}+\mathrm{Z}_{\mathrm{L} 2}\right) & =\alpha \cdot \mathrm{Z}_{\mathrm{L} 1} \\
\alpha & =\left(\mathrm{Z}_{\mathrm{R} 2} / \mathrm{Z}_{\mathrm{L} 2}+1\right)
\end{aligned}
$$

Thus, $\alpha$ must be set less than $\left[\left(Z_{\mathrm{R}_{2}} / \mathrm{Z}_{\mathrm{L} 2}\right)+1\right]$ or the amount of forward compensation $\left(\alpha \cdot \mathrm{Z}_{\mathrm{L} 2}\right)$ must not be set greater than $\mathrm{Z}_{\mathrm{R} 2}+\mathrm{Z}_{\mathrm{L} 2}$. This is an important observation for the next directional element method described.

## Relationship of Measured Z2 to Fault Direction

The compensated negative-sequence directional element uses voltage terms. These voltage terms are influenced by the measured $\mathrm{I}_{2}$. From Figure 9, the relay measures $\mathrm{I}_{\mathrm{S} 2}$ for forward faults and $\mathrm{I}_{\mathrm{R} 2}$ for reverse faults. Ohm's Law tells us that if we measure the voltage across an impedance, and the current through the same impedance, we can then calculate the impedance. From $\mathrm{V}_{2}$ and $\mathrm{I}_{2}$, calculate the measured negative-sequence impedance and call this impedance $\mathrm{Z} 2_{\text {MEASURED }}$ :

Forward SLG Faults: $\quad \mathrm{Z}_{\text {MEASURED }}=\mathrm{V}_{2} / \mathrm{I}_{\mathrm{S} 2} \quad=-\mathrm{Z}_{\mathrm{S} 2}$
Reverse SLG Faults: $\quad \mathrm{Z}_{\text {MEASURED }}=\mathrm{V}_{2} /\left(-\mathrm{I}_{\mathrm{R} 2}\right)=+\left(\mathrm{Z}_{\mathrm{L} 2}+\mathrm{Z}_{\mathrm{R} 2}\right)$
This relationship is shown in Figure 10 for a $90^{\circ}$ system.


Figure 10 Measured Negative-Sequence Source Impedance Yields Fault Direction
An earlier paper [1] describes a new negative-sequence directional element that measures z 2 and compares the result against simple thresholds to determine fault direction. The equation for this new negative-sequence directional element is shown below:

$$
\begin{equation*}
\mathrm{z} 2=\frac{\operatorname{Re}\left[\mathrm{V}_{2} \cdot\left(\mathrm{I}_{2} \cdot 1 \angle \Theta\right)^{*}\right]}{\left|\mathrm{I}_{2}\right|^{2}} \tag{6}
\end{equation*}
$$

where:
z2 = measured negative-sequence impedance.
$\angle \Theta=$ the angle of the negative-sequence line impedance.
The z2 of (6) is equivalent to $\mathrm{Z} 2_{\text {measured. }}$. The criteria for declaring forward and reverse fault conditions are:

- z2 $<$ Z2F threshold : Forward fault condition
- $\mathrm{z} 2>\mathrm{Z} 2 \mathrm{R}$ threshold : Reverse fault condition

Z2F must be less than Z2R to avoid any overlap where z2 satisfies both forward and reverse fault conditions. If the magnitude of the measured $V_{2}$ is so small as to give a z 2 result near zero, simply increase the Z2F threshold to gain directional security.

Recall from Figure 8 that $-\left(\alpha \cdot \mathrm{Z}_{\mathrm{L} 2} \cdot \mathrm{I}_{2}\right)$ increases the apparent magnitude of the negativesequence source impedance behind the relay location. This same task is accomplished by increasing the forward (Z2F) threshold.

## Combining Phase and Residual Currents in a Ground Directional Element

A ground directional element, which combines phase and residual currents, uses the polarizing and operating quantities shown in Table 6. Three separate ground directional elements are required: one for each phase.

Table 6 Inputs to a Simple Ground Directional Element

| Phase | Polarizing $V\left(\mathbf{V}_{\text {POL }}\right)$ | Operating Quantity $\left(\mathbf{V}_{\mathbf{O P}}\right)$ |
| :---: | :---: | :---: |
| A | $\mathrm{V}_{\mathrm{A}}+\mathrm{jV}_{\mathrm{BC}}$ | $\mathrm{Z}_{\mathrm{L} 1} \cdot\left(\mathrm{I}_{\mathrm{A}}+\mathrm{k} 0 \cdot 3 \mathrm{I}_{0}\right)$ |
| B | $\mathrm{V}_{\mathrm{B}}+\mathrm{jV}_{\mathrm{CA}}$ | $\mathrm{Z}_{\mathrm{L} 1} \cdot\left(\mathrm{I}_{\mathrm{B}}+\mathrm{k} 0 \cdot 3 \mathrm{I}_{0}\right)$ |
| C | $\mathrm{V}_{\mathrm{C}}+\mathrm{jV}_{\mathrm{AB}}$ | $\mathrm{Z}_{\mathrm{L} 1} \cdot\left(\mathrm{I}_{\mathrm{C}}+\mathrm{k} 0 \cdot 3 \mathrm{I}_{0}\right)$ |

This type of directional element has one major shortcoming: residual current is not phase selective, and a directional element for a non-faulted phase can produce errors.

Consider the system shown in Figure 11. Line 2 has an A-phase-to-ground fault with $1 \Omega$ of fault resistance near Bus S. Ideally, Relay 1 should declare the fault direction as reverse.


Figure 11 Reverse A-Ground Fault on Line 2 Close to Bus S
In the no load case $(\delta=0), \mathrm{I}_{\mathrm{B}}$ and $\mathrm{I}_{\mathrm{C}}$ are zero and all $\mathrm{V}_{\mathrm{OP}}$ quantities are in phase. Only the Bphase ground directional element declares the fault forward for $\delta=0$. As we increase load into and away from Bus S, the C-phase ground directional element also declares the reverse fault as forward (Figure 12).


Figure 12 With Combined Phase and Residual Currents in the Operating Quantity, Directional Element Declarations are Load Dependent

From Figure 12, the B-phase ground directional element always declares forward fault condition while the C-phase ground directional element decision depends upon load flow direction.

The inability of these three separate ground directional elements to agree on fault direction and their dependence upon load flow make these elements undesirable.

## Avoiding Directional Element Race Conditions

Consider the case of a ground directional element allowed to run unrestricted during non-fault conditions. This unrestricted directional element can pick up for normal system load unbalance. The direction declared by this directional element is not important until a ground fault occurs.

Next, suppose the power system in the immediate vicinity of the unrestricted directional element experiences a ground fault. If the load induced directional declaration is opposite that of the fault direction, there is a race between the pickup of the directional overcurrent element and the dropout of the load induced directional element decision (the directional element must first drop out from one directional declaration before declaring the opposite direction).

To avoid this race condition, the directional element should be controlled by an overcurrent element. You must set the sensitivity of this controlling overcurrent element above normal system unbalance. By requiring the controlling overcurrent element to pick up before the directional element is enabled, you avoid unbalanced load induced directional decisions.

## Directional Element Performance

## Negative-Sequence Directional Element in Series-Compensated Lines Applications

Figure 13 shows a 239.68 -mile long 345 kV transmission line with $42.5 \%$ line-end seriescompensation. Figure 14 shows the voltages and currents for a B-ground fault, located 108 miles from the relay location.


Figure 13 Faulted Power System. Transmission Line Length 239.68 Miles. Percentage of Series-Compensation 42.5


Figure 14 Voltages and Currents for a B-Ground Fault at 108 Miles from the Relay Location

Figure 15 shows the calculated z2 dynamic performance of the negative-sequence impedance directional element described earlier.

In Figure 15, we see that the calculated negative-sequence impedance ( z 2 ) has positive values at fault inception. For this fault, the calculated z 2 is always less than the forward Z2F threshold; $\mathrm{z} 2<\mathrm{Z} 2 \mathrm{~F}$ indicates a forward fault condition.

Appendix A shows the threshold setting calculations for this application.


Figure 15 Dynamic Response of the Negative-Sequence Directional Element for an Actual B-Phase-to-Ground Fault Condition

## Negative-Sequence Directional Element in Untransposed Line Applications

Untransposed transmission lines may have different self and mutual impedances. Three-phase faults on untransposed transmission lines generate positive-, negative-, and zero-sequence currents. An undesired negative-sequence directional element operation may occur due to the three-phase fault generated negative-sequence current.

The following values are the self and mutual impedances of a 400 kV , 110 kilometer long transmission line with bundled ACSR 1113 Blujay conductors. The 400 kV tower configuration is shown in Figure 16.

$$
Z_{L}=\left[\begin{array}{ccc}
Z a a & Z a b & Z a c \\
Z b a & Z b b & Z b c \\
Z c a & Z c b & Z c c
\end{array}\right]=\left[\begin{array}{ccc}
12.07+63.10 i & 9.25+22.08 i & 8.93+17.01 i \\
9.25+22.08 i & 12.44+62.00 i & 9.25+22.08 i \\
8.93+17.01 i & 9.25+22.08 i & 12.07+63.10 i
\end{array}\right]
$$

The self impedances (Zaa, Zbb, Zcc) and mutual impedances (Zab, Zba, Zac, Zac, Zbc, Zcb) are different because the distances between phase A, B, and C conductors are different.


Figure 16 Typical 400 kV Transmission Line Tower Configuration

Assuming zero source impedance and a radial system, Relay 1 measures the following sequence currents for a three-phase fault at the end of the 400 kV transmission line:

$$
\begin{array}{lll}
\mathrm{I}_{\mathrm{A} 0} & =74.21 & \angle-19.14^{\circ} \mathrm{Amps} \text {, primary. } \\
\mathrm{I}_{\mathrm{A} 1} & =5484.74 \quad \angle-85.84^{\circ} \text { Amps, primary. } \\
\mathrm{I}_{\mathrm{A} 2} & =486.26 \quad \angle 36.82^{\circ} \text { Amps, primary. }
\end{array}
$$

From these sequence currents, we see $\mathrm{I}_{\mathrm{A} 2} / \mathrm{I}_{\mathrm{A} 1}$ is small for three-phase faults. This same ratio approaches unity for phase-phase and phase-ground faults. We can use the $\mathrm{I}_{\mathrm{A} 2} / \mathrm{I}_{\mathrm{A} 1}$ ratio to supervise the negative-sequence directional element: enable the negative-sequence directional element if $\left|\mathrm{I}_{\mathrm{A}_{2}}\right| /\left|\mathrm{I}_{\mathrm{A} 1}\right| \geq \mathrm{a} 2$. Call this a 2 scalar the positive-sequence restraint factor.

For the untransposed 400 kV transmission line described above, calculate the minimum a2 factor:

$$
\mathrm{a} 2=\left|\mathrm{I}_{\mathrm{A} 2}\right| /\left|\mathrm{I}_{\mathrm{A} 1}\right|=486.26 \mathrm{~A} / 5484.74 \mathrm{~A}=0.09
$$

This positive-sequence restraint factor should be settable to adapt to any line configuration. In this 400 kV application, setting a2 $>0.09$ prevents the negative-sequence directional element from operating for three-phase faults.

## Negative-Sequence Impedance vs. Positive-Sequence Impedance Directional Element

McLaren et al. [5] proposes a new directional element for determining fault direction, regardless of fault type. This proposed directional element first calculates $Z_{1}$, the positive-sequence impedance, and then tests the result with impedance plane quadrant checks to determine fault direction.

If the calculated $\mathrm{Z}_{1}$ value resides in the third quadrant in the impedance plane, the fault is considered forward. If the calculated $\mathrm{Z}_{1}$ value is in the first quadrant, the fault is considered reverse.

The following equation shows the $\mathrm{Z}_{1}$ calculation:

$$
\mathrm{Z}_{1}=\left(\mathrm{V}_{1}-\mathrm{V}_{1 \text { PRE }}\right) /\left(\mathrm{I}_{1}-\mathrm{I}_{\text {IPRE }}\right)
$$

where:

$$
\begin{aligned}
\mathrm{V}_{1} & =\text { Positive-sequence fault voltage. } \\
\mathrm{V}_{\text {IPRE }} & =\text { Positive-sequence prefault voltage. } \\
\mathrm{I}_{1} & =\text { Positive-sequence fault current. } \\
\mathrm{I}_{1 \text { PRE }} & =\text { Positive-sequence prefault current. }
\end{aligned}
$$

Table 7 compares the performance of the new negative-sequence impedance and the positivesequence directional elements for the fault location shown in Figure 17. Figure 17 shows the two-source system model used to obtain the results in Table 7.


Figure 17 Two-Source System Model to Analyze $Z_{1}$ and Negative-Sequence Directional Element Performance for Different Fault Conditions

From Table 7, we see that both directional elements perform well for the shown fault conditions. However, the positive-sequence directional element requires valid prefault voltage and current data to make the correct directional decision.

Table 7 Calculated z 2 and $\mathrm{Z}_{1}$ Values for Different Fault Conditions

| Fault | $\boldsymbol{\delta}$ | $\mathbf{z 2}$ | $\mathbf{Z}_{\mathbf{1}}$ |
| :---: | :---: | :---: | :---: |
| A-G | $0^{\circ}$ | -0.8 | $0.8 \angle-90$ |
| B-C | $0^{\circ}$ | -0.8 | $0.8 \angle-90$ |
| B-C-G | $0^{\circ}$ | -0.8 | $0.8 \angle-90$ |
| A-G | $30^{\circ}$ | -0.8 | $0.8 \angle-90$ |
| B-C | $30^{\circ}$ | -0.8 | $0.8 \angle-90$ |
| B-C-G | $30^{\circ}$ | -0.8 | $0.8 \angle-90$ |
| A-G | $-30^{\circ}$ | -0.8 | $0.8 \angle-90$ |
| B-C | $-30^{\circ}$ | -0.8 | $0.8 \angle-90$ |
| B-C-G | $-30^{\circ}$ | -0.8 | $0.8 \angle-90$ |

The negative-sequence directional element calculation is the same for all faults applied, does not require prefault data, and does not depend on load conditions. A simple threshold check suffices to determine if the fault is in the forward or reverse direction.

## Summary

Important points presented in this paper include the following:

1. Replacing the traditional collection of three-phase directional elements with positive-sequence and negative-sequence directional elements improves security.
2. Positive-sequence directional elements must include polarizing memory to assure operation for close-in, three-phase faults.
3. Zero-sequence mutual coupling adversely affects zero-sequence polarized ground directional elements in applications where the zero-sequence sources are isolated.
4. A negative-sequence directional element that calculates negative-sequence impedance and tests the result against thresholds is presented. This directional element is applicable to most systems. Load conditions do not affect the negative-sequence directional element calculation. A simple threshold check suffices to determine fault direction. Minimum voltage sensitivity is not a concern with this improved directional element.
5. Settable forward and reverse thresholds for the negative-sequence impedance directional element increase the scope of applications. For example, set Z2F greater than zero in seriescompensated line applications with line-side voltage transformers.
6. We point out a problem with ground directional elements that use residual current in the operating quantity and phase voltage combinations as the polarizing quantity.
7. Supervising directional calculations with overcurrent elements avoids decisions based upon unbalanced load.

In conclusion, discrete directional elements have poor performance if they are allowed to act on their own for all fault types. Merging all directional elements into one numerical relay hardware package permits the microprocessor to determine which directional decision is best.

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## APPENDIX A: NEGATIVE-SEQUENCE DIRECTIONAL ELEMENT THRESHOLD SETTING CALCULATIONS

The negative-sequence secondary impedances of the system shown in Figure A1 are:

$$
\begin{array}{ll}
\text { Strongest local source: } & \mathrm{Z}_{\mathrm{S} 2}=\mathrm{j} 2.9 \Omega \\
\text { Series capacitor: } & \mathrm{Z}_{\mathrm{C}}=-\mathrm{j} 7.6 \Omega \\
\text { Transmission line: } & \mathrm{Z}_{\mathrm{L} 2}=\mathrm{j} 17.9 \Omega \\
\text { Strongest remote source: } & \mathrm{Z}_{\mathrm{R} 2}=\mathrm{j} 1.8 \Omega
\end{array}
$$

In this application, the voltage transformers are located on the line-side of the series-capacitors. For this voltage transformer location, the series-capacitor impedance must be included in the negative-sequence impedance calculation for forward faults.

The negative-sequence impedance calculation for forward faults is:

$$
\mathrm{z} 2 \mathrm{~F}=-\left(\mathrm{Z}_{\mathrm{S} 2}+\mathrm{Z}_{\mathrm{C}}\right)=-(\mathrm{j} 2.9-\mathrm{j} 7.6)=\mathrm{j} 4.7
$$

The calculation for reverse faults is:

$$
\mathrm{z} 2 \mathrm{R}=\mathrm{Z}_{\mathrm{R} 2}+\mathrm{Z}_{\mathrm{L} 2}=\mathrm{j} 1.8+\mathrm{j} 17.9=\mathrm{j} 19.7
$$

Set the forward and reverse thresholds between the forward and reverse fault calculation results ( z 2 F and z 2 R ). The difference between z 2 F and z 2 R defines the setting region. The forward threshold (Z2F) must be set above z2F and the reverse threshold (Z2R) below z2R.

$$
Z_{\text {REGION }}=\mathrm{z} 2 \mathrm{R}-\mathrm{z} 2 \mathrm{~F}=\mathrm{j} 19.7-\mathrm{j} 4.7=\mathrm{j} 15.0
$$

The following expressions determine the forward and reverse thresholds:

$$
\begin{array}{ll}
\mathrm{Z} 2 \mathrm{~F}=\mathrm{z} 2 \mathrm{~F}+\left(\mathrm{Z} 2_{\text {REGION }}\right) / 3=\mathrm{j} 4.7+\mathrm{j} 5.0=\mathrm{j} 9.7, & \mathrm{Z} 2 \mathrm{~F}=9.7 \\
\mathrm{Z} 2 \mathrm{R}=\mathrm{z} 2 \mathrm{R}-\left(\mathrm{Z} 2_{\text {REGION }}\right) / 3=\mathrm{j} 19.7-\mathrm{j} 5.0=\mathrm{j} 14.7, & \mathrm{Z} 2 \mathrm{R}=14.7
\end{array}
$$

Figure A1 shows the negative-sequence impedance plane with forward and reverse thresholds and relay calculations for forward and reverse faults.


Figure A1 Settings and Relay Calculated Impedances in the Negative-Sequence Impedance Plane

The calculated impedance ( z 2 F ) is less than the Z2F setting for forward faults, and the calculated impedance ( z 2 R ) is greater than the Z2R setting for reverse faults.

