Capacitor Bank Unbalance Protection Calculations and Sensitivity Analysis

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***Abstract*—In this paper, we introduce a method for performing unbalance calculations for high-voltage capacitor banks. We consider all common bank configurations and fusing methods and provide a direct equation for the operating signal of each of the commonly used unbalance protection elements. This one-step calculation method requires less data and is not only simpler but also less prone to errors compared with multistep methods such as the ones included in the IEEE Std C37.99 [1]. Our equations cover both the fail-open and fail-short failure scenarios (fused, fuseless, and temporarily repaired banks). The paper also derives equations for calculating the degree of internal overvoltage that a failure puts on the healthy capacitor units in the bank. Next, we derive equations for the unbalance protection operating signals as functions of the internal overvoltage. Our equations tie together the unbalance protection operating signals, the number of failed capacitor units, and the internal overvoltage caused by the failure. Therefore, these equations provide a solid basis for setting the unbalance protection elements: we set the alarm thresholds to detect a single (or partial) unit failure, and we set the trip thresholds to trip when the internal overvoltage caused by the failure threatens a cascading failure. The paper includes dozens of equations because it covers a variety of bank configurations, fusing methods, and unbalance protection elements. However, when working on a particular bank configuration with a particular fusing method, the user is presented with just a handful of simple equations for direct one-step calculation of the unbalance protection operating signals.**

I. Introduction

Shunt capacitor banks are assembled from capacitor units connected in parallel to form groups, groups connected in series to form strings, and stings connected in parallel to form phases. In high-voltage applications, the phases are connected as grounded or ungrounded single-wye, double-wye, or H-bridge bank configurations. Capacitor units, in turn, are fabricated from capacitor elements encased together and connected in parallel-series structures. Fuses may be applied to address failures of capacitor elements (internally fused banks) or units (externally fused banks). The method of fusing impacts how the capacitor units are arranged in groups and strings.

Overall, capacitor banks are protected by a combination of fuses, which remove the failed unit or element, and protective relays, which alarm and trip the bank offline. From the protective relay perspective, a capacitor failure and the resulting fuse operation (if fuses are used) blend together into a single event to be detected (alarm for failures that can be tolerated and trip for failures that may progress catastrophically because of the overvoltage condition that the failure puts on the remaining healthy capacitor units).

Capacitor failures cause only slight changes in the bank voltages and currents. Therefore, these failures cannot be detected based on the levels of voltages and currents but based on the unbalance in voltages and currents relative to a healthy bank (hence the name unbalance protection). A distinct set of unbalance protection elements is available for each bank configuration.

To set the unbalance protection elements, we must perform fault calculations for series failures inside the capacitor bank (capacitor units or elements failing open or short). Because capacitor bank equations are linear and there is no mutual coupling inside the bank, the underlying equations for the calculations are simple: the unit reactance ties the unit voltage and current while Kirchhoff's laws tie all voltages and currents inside the bank. However, solving these underlying equations by hand is tedious.

In general, we use short-circuit programs to set protection elements, such as distance or overcurrent. However, the commonly used short-circuit programs do not include modules for unbalance calculations in capacitor banks. The IEEE Std C37.99 [1] advocates numerical multistep unbalance calculations. Often, about a dozen calculation steps are required to obtain an unbalance protection element operating signal. Some users develop their own short-circuit programs for unbalance calculations in capacitor banks. Developing and validating these specialized short-circuit programs is time consuming.

This paper fills this void and provides equations for unbalance calculations for common bank configurations, fusing methods, and unbalance protection elements. These equations allow direct (one-step) calculation because they directly tie the unbalance protection operating signals to the capacitor unit arrangement parameters and the size (number of failed units), type (fail-open or fail-short), and location (above or below the bridge, left or right half of the bank, phase A, B, or C) of the failure. Avoiding multistep calculations not only reduces time and effort but also eliminates opportunities for errors.

The unbalance protection equations are remarkably simple. We achieved this simplicity by working in per-unit values. It is apparent that an unbalance in capacitor bank voltages and currents is a result of a difference between the faulted and healthy parts of the bank. As such, the per-unit voltage or current unbalance is independent of the absolute characteristics of the faulted and healthy parts. We will show that the unbalance in per unit is a fractional number: a ratio of two integer numbers that depend on the number of failed capacitor units and the number of units, groups, and strings in the bank.

We introduce the concept of an overvoltage factor. We define it as a ratio of the voltage elevated by the failure in the most stressed part of the bank and the normal voltage in that part. The overvoltage factor is a simple function of the bank parameters and the size, type, and location of the failure. We use the overvoltage factor to better understand the impact of a failure on the rest of the bank including the danger of breaching the unit voltage rating and causing a cascading failure. More importantly, we use the overvoltage factor to set unbalance protection elements. We propose setting the alarm threshold to detect a single unit failure (or even a fractional unit failure because of the failure of some but not all capacitor elements inside the unit) and setting the trip threshold to trip before the internal overvoltage caused by the failure exceeds the unit voltage rating and triggers a cascading failure.

Unlike the numerical solutions (numbers in, numbers out), our analytical equations directly tie the signals of interest to the failure and bank parameters. As a result, these direct equations allow a multitude of applications and insights.

The paper is organized as follows.

Section II reviews the common high-voltage capacitor bank configurations and the applicable unbalance protection elements. The section states the scaling and measuring polarity conventions for the unbalance protection elements.

Section III explains the capacitor unit arrangement that we assumed when deriving our equations, explains the capacitor failure scenarios, and introduces the per-unit system for the calculations. The capacitor unit arrangement assumed in this paper covers most practical cases. The failures include the fail-open and fail-short scenarios. You can use these failure scenarios to represent failures in the fused and fuseless banks as well as temporary repairs in the bank (leaving the failed units open or shorted until a proper repair can be performed).

Section IV uses an example to explain how we derived and validated the equations.

Section V introduces the concept of the overvoltage factor and explains how to use it to set trip thresholds for the unbalance protection elements.

Section VI provides several unbalance protection settings calculation examples to better explain and illustrate the new concepts.

Section VII gathers insights from the derived equations. It points to similarities between various unbalance protection elements and compares their relative sensitivity.

Appendix A is a compilation of the derived equations. While the paper explains our methodology and teaches how to use the new information, Appendix A is a key output of our work. The appendix is formatted for ease of use and reproduction in your project documentation. To appreciate the output of this paper, consider looking at Appendix A before reading the remainder of the paper.

Appendix B addresses the issues of multiple bank failures and shows how to use our equations to leverage the principle of superposition and perform unbalance calculations for multiple failures occurring sequentially in different parts of the bank.

Appendix C shows how to use our equations to perform unbalance calculations for capacitor element failures by treating them as partial capacitor unit failures.

II. Bank Configurations and Unbalance Protection

We consider the following common configurations of high-voltage capacitor banks:

- Grounded single-wye
- Ungrounded single-wye
- Grounded double-wye
- Ungrounded double-wye
- Grounded H-bridge
- Ungrounded H-bridge

with the following fusing methods:

- Externally fused
- Internally fused
- Fuseless

Advantages, drawbacks, and application considerations for each of these bank configurations and fusing methods are out of the scope of this paper.

We consider the following unbalance protection elements:

- Neutral overvoltage (59NT) for ungrounded banks
- Neutral overvoltage unbalance (59NU) for ungrounded banks
- Voltage differential (87V) for grounded banks and ungrounded double banks
- Neutral overcurrent unbalance (60N) for grounded and ungrounded double banks
- Phase overcurrent unbalance (60P) for grounded and ungrounded double banks
- Negative-sequence overcurrent (50Q/50QT) for grounded and ungrounded banks
- Impedance (21C) for grounded banks

Fig. 1 through Fig. 6 show the bank configurations and the applicable unbalance protection elements. The figures show the unbalance protection elements applicable to each bank configuration by denoting their ANSI device numbers (in green) and the unbalance protection operating signal names and measuring conventions (in blue). For simplicity, the figures do not show instrument transformers, unless required for clarity. The figures also show the failure location for which the derived unbalance equations directly apply. The term location refers to the faulted phase (A, B, or C), the faulted half of the double bank (left or right), and the fault position with respect to the tap or bridge (above or below).

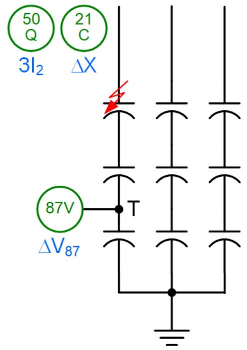


Fig. 1. Grounded single-wye bank configuration and unbalance protection.

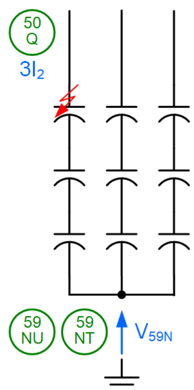


Fig. 2. Ungrounded single-wye bank configuration and unbalance protection.

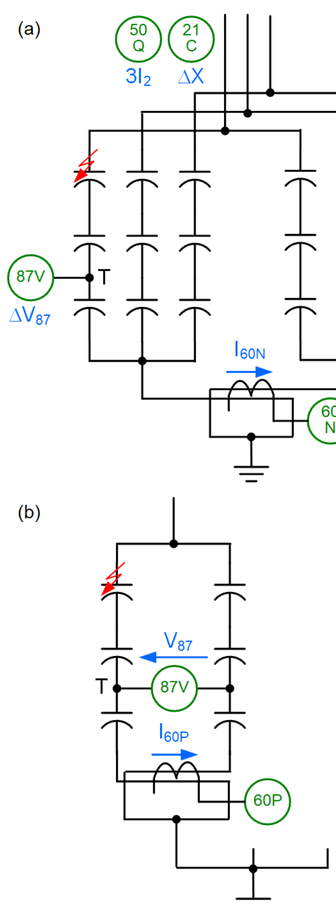


Fig. 3. Grounded double-wye bank configuration and unbalance protection (a) and 60P protection and alternative connection of the 87V protection (b).

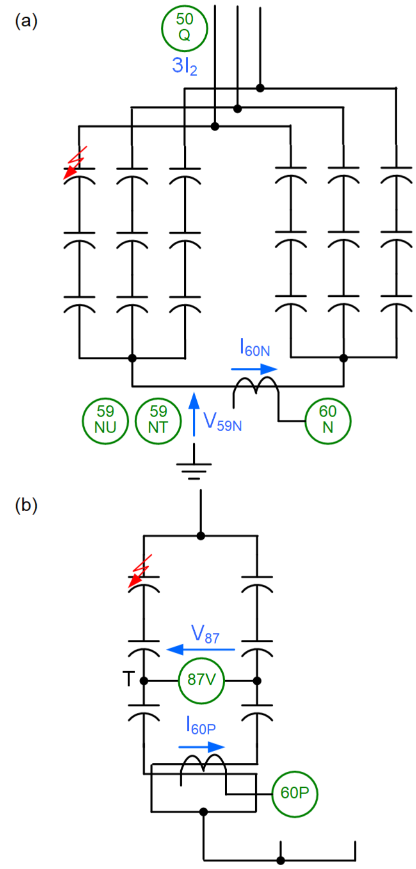


Fig. 4. Ungrounded double-wye bank configuration and unbalance protection (a) and 60P protection and alternative connection of the 87V protection (b).

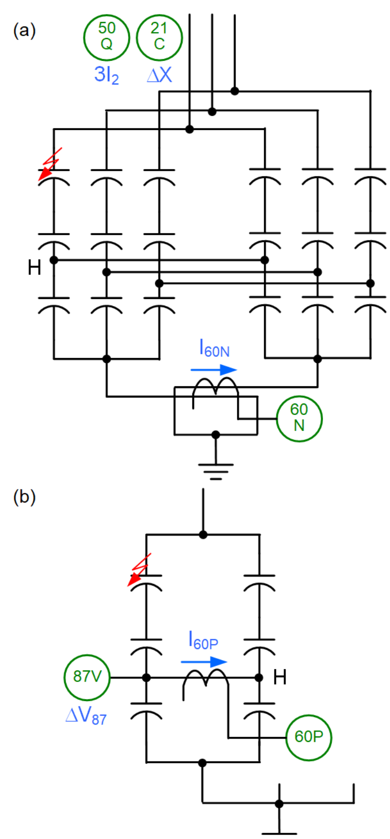


Fig. 5. Grounded H-bridge bank configuration and unbalance protection (a) and 60P and 87V protection (b).

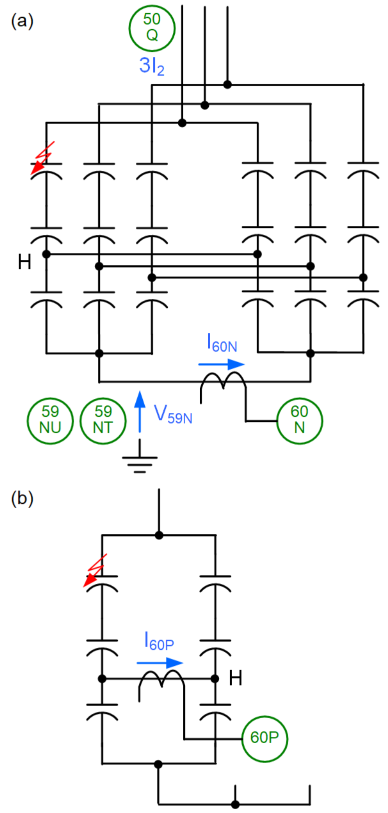


Fig. 6. Ungrounded H-bridge bank configuration and unbalance protection (a) and 60P protection (b).

To understand the scaling and polarity conventions for the protection input signals, consider the following points.

We define the operating signal for the 59NU element as follows:

$$V_{59NU} = V_N - 3V_0 \tag{1}$$

where V_N is the bank neutral-point voltage and $3V_0$ is the tripled zero-sequence bus voltage.

When only a small number of capacitor units fail, the bus voltages remain balanced ($3V_0 = 0$), and using (1) makes the operating signals of the 59N/59NT and 59NU elements equal during bank failures:

$$V_{59NU} = V_{59N} = V_N \tag{2}$$

As a result, we only need to provide unbalance calculations for the neutral-point voltage, and during capacitor unit failures, these calculations apply to both the 59NT and 59NU protection elements. Of course, the 59NT element uses time delay for security during system faults, while the 59NU element can operate with little or no time delay. We refer to the operating signal of the 59NT and 59NU elements that is common during series faults as V_{59N}.

To simplify analysis and calculations for the two possible connections of the voltage differential protection element (compare Fig. 1 and Fig. 3b), we use the following 87V operating signal convention:

$$\Delta V_{87} = V_{TAP} - T \cdot V_{BUS} \tag{3}$$

where T is the per-unit tap position (the reactance between the tap and the neutral point of the bank in per unit of the bank reactance).

Using (3), the operating signal of the voltage differential element is independent of the way of obtaining the reference signal ($T \cdot V_{BUS}$). The ΔV_{87} signal obtained by measuring the bus voltage (Fig. 1) and the V_{87} signal obtained by measuring the voltage difference between taps in a double bank (Fig. 3b) are equal:

$$V_{87} = \Delta V_{87} \tag{4}$$

As a result, we only need to provide one version of the 87V unbalance equation, and the equation applies to both ways of connecting the 87V element. If your 87V application is based on a different tap matching convention than (3), then rescale the 87V operating signal obtained by using this paper.

Typically, the 87V tap position T is considerably less than 0.5 pu. For H-bridge banks (Fig. 5b), we use variable H for the per-unit position of the bridge and assume that the 87V tap position T is the same as the bridge position H. Typically, H is about 0.5 pu.

We show the neutral (60N) and phase (60P) current unbalance protection elements connected to low-ratio window current transformers (CTs). These CTs measure the unbalance currents (difference between two currents) through magnetic summation of the two fluxes. This measurement method is considerably more accurate than the method of using two differentially connected CTs to sum the secondary currents. When using window CTs, the 60P operating signal balances without errors during normal bank operation and external faults, and the 60N operating signal balances without errors during system ground faults.

The 50Q protection uses breaker CTs and measures the negative-sequence current ($3I_2$) at the bank terminals. For double banks, the 50Q element measures the total bank current. Of course, when set to detect capacitor unit failures, the 50Q element uses time delay (50QT) for security during system faults. For simplicity and to avoid considerations regarding inverse-time or definite-time delay, we refer to this unbalance protection element as 50Q in the context of its operating signal.

We consider the 21C protection only for grounded banks. We provide unbalance calculations (apparent reactance change) for both the per-phase and per-string applications of the 21C protection. The latter approach offers significantly better sensitivity but requires multiple CTs per phase (to reduce their voltage rating and cost, these CTs are installed in the bottom of the bank near the ground potential). For the grounded H-bridge banks, we consider only the per-phase 21C element.

From the traditional protection perspective (alarm and trip thresholds), phase angles of the unbalance protection operating signals are irrelevant. In this paper, however, we pay attention to the instrument transformer polarity convention and the unbalance protection operating signal phase angle. Treating the unbalance signals as phasors (considering both the magnitude and angle) facilitates the application of the principle of superposition in unbalance calculations for multiple bank failures (Appendix B).

We use the following measurement and angle conventions:

- Unbalance currents (I_{60N} and I_{60P}) are measured away from the failed half of the bank.
- Unbalance voltages are measured as a potential of the failure area of the bank respective to ground (V_{59N}), the bus (ΔV_{87} and V_{59NU}), or the tap in the healthy half of the double bank (V_{87}).
- Phase angles of all the unbalance protection operating signals are referenced to the faulted-phase voltage angle.
- Apparent impedance is calculated as the phase-to-ground capacitive reactance, i.e., a real positive value.

Phase angles of the unbalance protection operating signals are also useful when looking for the location of the failure inside the bank: Which phase has a failure? Is the failure above or below the tap or bridge point? Is the failure in the left or right half of the bank? See [2] for more information on fault locating in capacitor banks.

III. Bank Unit Arrangement and Failure Scenarios

A. Capacitor Unit Arrangement

Fig. 7 shows a general arrangement of capacitor units in a single phase (Φ) of a high-voltage capacitor bank. P capacitor units are connected in parallel to form a group. S groups are connected in series to form a string. R strings are connected in parallel to form a phase. In fuseless banks, P = 1, and in fused banks, R is typically 1 or 2.

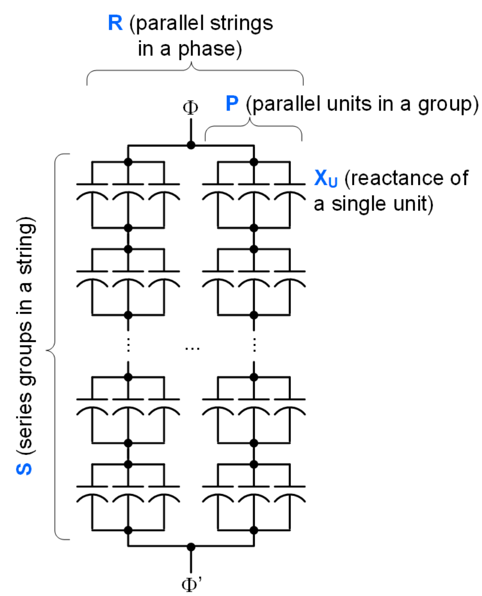


Fig. 7. General capacitor unit arrangement in the capacitor bank phase.

Typically, S is much greater than 1 because the unit voltage rating is a relatively small fraction of the system nominal voltage, and several units must be connected in series to match the system voltage. The product of S, R, and P is much greater than 1 to provide the desired rated power given the unit reactive power rating (kVAr). Often, the sum of R and P is also much greater than 1 to increase bank survivability during unit failures, even though P or R can be as low as 1.

The phase reactance (X) is the following function of the capacitor unit reactance (X_U) and the bank parameters S, P, and R:

$$X = \frac{S}{P \cdot R} \cdot X_U \tag{5}$$

We assume a uniform bank unit arrangement as follows:

- All phases (A, B, and C) are constructed the same.
- Both halves of a double bank are constructed the same.
- In H-bridge banks, the parts above and below the bridge are constructed with the same capacitor units and the same P and R parameters, even though the number of groups in series may be different above and below the bridge (the bridge position H may be different than 0.5 pu; S is the total number of groups in series).

We recognize that when an 87V tap is created, the number, arrangement, and ratings of capacitor units are often different in the parts above and below the 87V tap. Typically, the top (high voltage) part serves to provide reactive power and the bottom (low voltage) part acts only as a reference for the 87V protection element (Fig. 8). The reactive power of the low-voltage part is typically two orders of magnitude lower than the high-voltage part and can be neglected.

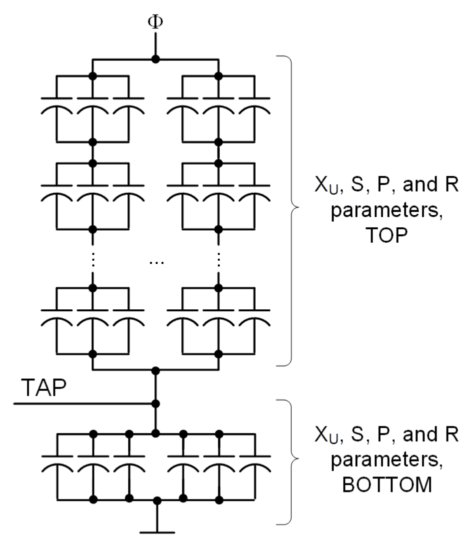


Fig. 8. Nonuniform unit arrangement when creating an 87V tap.

The per-unit tap position T follows the voltage divider principle. We calculate T as follows:

$$T = \frac{X_{BOTTOM}}{X_{TOP} + X_{BOTTOM}} \tag{6}$$

where X_{TOP} and X_{BOTTOM} are the reactances of the top and bottom parts, respectively (use (5) to calculate these reactances).

To remove this tap-related nonuniformity and use a uniform unit arrangement in all our calculations, we introduce and use the equivalent bank parameters as follows:

- The equivalent number of parallel units in a group, P, is the same as in the top (above the tap) part of the phase.
- The equivalent number of parallel strings in a phase, R, is the same as in the top (above the tap) part of the phase.
- The equivalent number of groups in series in a string is:

$$S_{EQ} = \frac{S_{TOP}}{1 - T} \tag{7}$$

Typically, (7) returns the equivalent value of S that is not an integer number. The noninteger value of S does not create any problems in subsequent unbalance calculations but allows us to neglect the differences in unit arrangement in the parts above and below the tap.

<u>Example 1</u>

A grounded single-wye capacitor bank is constructed with one 580 kVAr, 17.5 kV capacitor unit per group, 8 groups in series in a string, and 6 strings in parallel per phase. The low-voltage tap is created by using two 167 kVAr, 825 V capacitor units connected in parallel. We calculate the reactance values of the top and bottom capacitor units from their rated power and voltage and obtain 528.017 Ω and 4.0756 Ω, respectively.

We use (5) and calculate the reactance values of the parts above and below the tap:

$$X_{TOP} = \frac{8}{1 \cdot 6} \cdot 528.017\ \Omega = 704.023\ \Omega$$

$$X_{BOTTOM} = \frac{1}{1 \cdot 2} \cdot 4.0756\ \Omega = 2.038\ \Omega$$

We use (6) and calculate the per-unit tap position:

$$T = \frac{2.038\ \Omega}{704.023\ \Omega + 2.038\ \Omega} = 0.002886\ \text{pu}$$

We use (7) and calculate the equivalent number of series groups, S:

$$S_{EQ} = \frac{8}{1 - 0.002886} = 8.023$$

Now we can neglect the nonuniformity of the capacitor unit arrangement and treat the bank as uniformly constructed with the following equivalent parameters: P = 1, S = 8.023, R = 6, and T = 0.002866 pu.

B. Failure Scenarios

High-voltage shunt capacitor banks fall into the following three categories:

- Externally fused banks where fuses mounted outside each capacitor unit case protect the bank from short circuits inside the units by disconnecting the shorted units.
- Internally fused banks where multiple “simplified” fuses are fabricated inside each capacitor case to protect the capacitor unit from short circuits of the capacitor elements inside the case.
- Fuseless banks where no fuses are present and the capacitor failures are permanent short circuits.

In general, we need to consider two general categories of failures: capacitors *failing open* (a short circuit blows the fuse and the faulted capacitance becomes an open circuit) and capacitors *failing short* (no fuse is present and the faulted capacitance remains shorted out). We will use fail-open and fail-short categories to represent temporary repairs in the bank.

In general, we can look at failures from the perspective of a capacitor unit or a capacitor element. A failure of a single capacitor element, or even a few elements, does not necessarily result in the loss of the entire capacitor unit. From this perspective, ability to perform unbalance calculations for a partial unit failure is beneficial. It allows analyzing the following cases:

- A few capacitor elements fail short in a capacitor unit of an externally fused bank, but the current that the failed unit draws is below the fuse rated current, and the unit does not fail open through the operation of the external fuse.
- A few capacitor elements fail open in an internally fused capacitor unit (through the operation of internal fuses), but the entire unit does not become an open circuit yet.
- A few capacitor elements fail short in a fuseless bank. These failures could short the entire capacitor unit, less than one capacitor unit, or more than one capacitor unit.

Typically, capacitor element failures put the highest voltage stress on the rest of the failed capacitor unit. As a result, the failure progresses first inside the unit, resulting in a complete failure of the unit before the problem spreads to the rest of the bank.

Ideally, to obtain better resolution of our calculations, we should perform all derivations from the perspective of the capacitor elements rather than the capacitor units. However, such an approach would be complicated for at least the following reasons: 1) for externally fused banks, we would see combinations of the fail-short (element failures) and fail-open (blown unit fuse) scenarios, 2) in our calculations, we will need to consider fuses that protect various numbers of capacitors such as external fuses protecting the entire capacitor unit and internal fuses protecting one or a few capacitor elements, and 3) the structure of the bank (how the capacitor units are arranged in the bank) and the structure of the unit (how the capacitor elements are arranged in the capacitor unit) would create a very complicated overall structure of the bank to analyze.

We solve this challenge as follows:

- We approach the calculations from the perspective of a capacitor unit and assume a failure (open or short) of the entire unit.

- We derive the unbalance equations by assuming an integer number of units that failed open or short (complete failure, not a partial failure).
- We devise a method to represent a partial capacitor unit failure by using a fractional number of failed units. This allows us to apply the unbalance equations derived for capacitor unit failures to complex failure scenarios with capacitor units and capacitor elements failing in arbitrary combinations.

In this paper, we use two generic capacitor unit failure categories: fail-open and fail-short.

The first category (fail-open) applies to both externally fused banks, in which the fuse operation removes the entire unit, and internally fused banks, when enough internal fuses operate to remove the entire unit. A common assumption is that the fuse operation is so fast that no unbalance protection element detects the shorted state. Therefore, the unbalance protection is concerned with detecting the resulting fail-open state after the fuse operation has removed the shorted unit.

The second category (fail-short) applies mainly to fuseless banks in which a short circuit inside the unit prevails and the unbalance protection is expected to detect the shorted unit(s). This category also applies to externally fused banks if you want to perform unbalance calculations at the capacitor element level for scenarios where some elements are shorted but the unit remains in service because the current of the failed unit is below the rated current of the external fuse.

We also use the fail-open and fail-short failure categories to represent banks that are left unrepaired or are temporarily repaired after a failure. In this application, we use the unbalance equations to calculate the inherent (standing) unbalance before the next failure, as well as to calculate the unbalance protection operating signals for a failure in an inherently unbalanced bank. Addressing the inherent unbalance in capacitor bank protection is out of the scope of this paper (see [3] for more details).

Fig. 9 shows our unit failure model for the fail-open scenario. When the first unit in a group fails open, the other units in the same group are subjected to an overvoltage condition. As a result, it is most likely that the next unit failure will be in the same group. Therefore, in the fail-open scenario, we assume that F units in the same group failed open (F < P).

Fig. 10 shows our unit failure model for the fail-short scenario. When the first unit in a group fails short, the other groups in the same string are subjected to an overvoltage condition. As a result, it is most likely that the next failure will be in a different group of the same string. Therefore, in the fail-short scenario, we assume that F groups in the same string failed short (F < S).

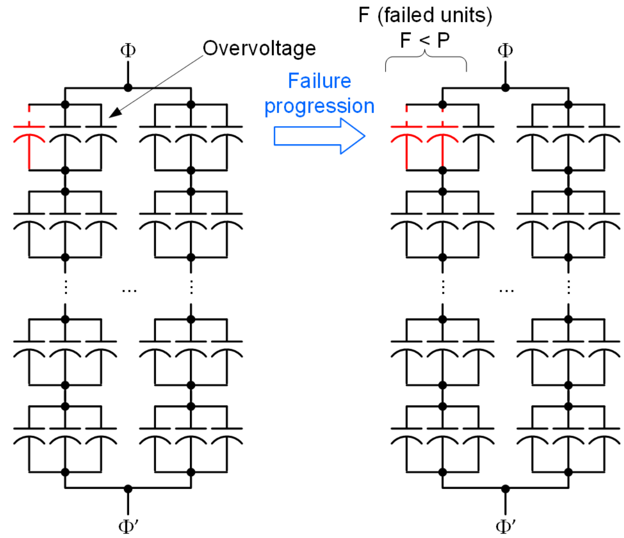


Fig. 9. Fail-open unit failure model.

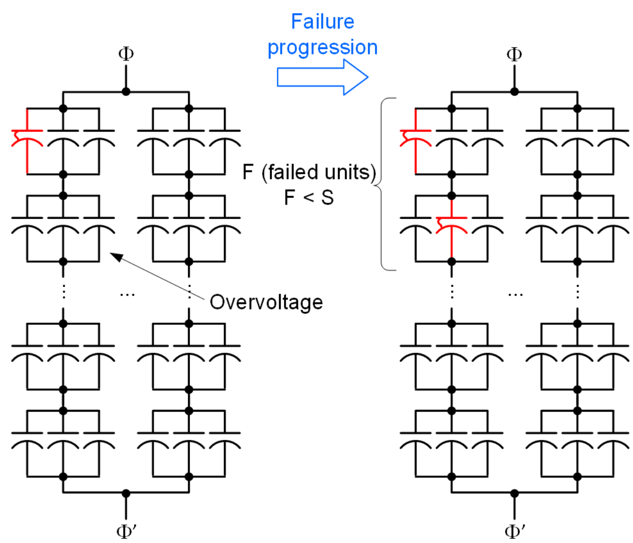


Fig. 10. Fail-short unit failure model.

In Section IV, we use variable F to represent the size of a failure that occurred in a single location, as shown in Fig. 9 and Fig. 10. In Appendix B, we use the superposition principle to perform unbalance calculations for failures at two or more different locations in the bank. Finally, in Appendix C, we use a fractional value of F to perform unbalance calculations for capacitor element failures.

C. *Per-Unit System*

Because equations that tie the capacitor bank voltages and currents are linear, we can select any unit convention for the capacitor bank calculations. By working in per unit, we follow a long tradition of short-circuit calculations in electric power systems. We will see that performing calculations in per unit of the bank nominal voltage and current yields simple results that are applicable to banks of any voltage (V_{NOM}) and reactive power (Q_{NOM}) ratings.

We use the following base quantities. The base voltage is the nominal phase-to-ground bank voltage:

$$V_{BASE} = \frac{V_{NOM}}{\sqrt{3}} \tag{8}$$

We selected the phase-to-ground voltage because the phase-to-ground voltage is applied to the phase reactance in the wye-connected capacitor banks covered in this paper (we do not consider delta-connected banks).

The base current is the nominal bank current:

$$I_{BASE} = \frac{Q_{NOM}}{\sqrt{3} \cdot V_{NOM}} = \frac{Q_{NOM}}{3 \cdot V_{BASE}} \tag{9}$$

Q_{NOM} is the reactive power that the bank provides under nominal voltage, not the sum of unit power ratings (kVAr) of all the capacitor units in the bank. In double-bank configurations, the base current is the nominal current of the entire bank, not half the bank.

Of course, the base reactance is:

$$X_{BASE} = \frac{V_{BASE}}{I_{BASE}} \tag{10}$$

In this per-unit system, the healthy-phase reactance is 1 pu and the faulted-phase reactance is slightly above or slightly below 1 pu depending on if the units fail open or short.

We will see that the unbalance calculations performed in this per-unit frame involve only the failure parameters (F, fail-open or fail-short) and the bank parameters (P, S, R, and H or T).

We perform unbalance calculations in the per-unit frame and obtain the unbalance protection operating signals. When converting the unbalance protection operating signals from per-unit values (PU) to secondary values (SEC), we apply the following unit conversions.

Voltage unbalance protection elements:

$$V_{SEC} = V_{PU} \cdot \frac{V_{BASE}}{PTR} \tag{11}$$

Current unbalance protection elements:

$$I_{SEC} = I_{PU} \cdot \frac{I_{BASE}}{CTR} \tag{12}$$

Reactance unbalance protection elements:

$$X_{SEC} = X_{PU} \cdot X_{BASE} \cdot \frac{CTR}{PTR} \tag{13}$$

where PTR and CTR are, respectively, the ratios of the voltage and current transformers providing signals to the associated protection element.

IV. Capacitor Bank Unbalance Calculations

In this section, we explain how we derived and validated the unbalance calculation equations listed in Appendix A.

A. Principles

We start with first principles (Kirchhoff's voltage and current laws and the unit reactance that ties the unit voltage and current); write a set of high-level equations for a given bank configuration, failure scenario, and unbalance protection element; and solve these equations in the per-unit frame for the unbalance protection element operating signal of interest.

We will use the 59N element for an ungrounded single-wye bank as an example. In this case, the Kirchhoff's current law is the starting point (in an ungrounded bank, the phase currents sum up to zero; see Fig. 2):

$$\frac{V_A - V_{59N}}{-jX_F} + \frac{V_B - V_{59N}}{-jX} + \frac{V_C - V_{59N}}{-jX} = 0 \tag{14}$$

where X_F and X are reactances of the faulted phase A, and the healthy phases B and C, respectively.

A failure involving a few capacitor units does not change the bus voltages, and therefore, we can substitute:

$$V_B = a^2 \cdot V_A, \qquad V_C = a \cdot V_A, \qquad a = 1\angle 120° \tag{15}$$

Inserting (15) into (14) and simplifying, we obtain:

$$V_{59N} \cdot \left(\frac{1}{X_F} + \frac{2}{X}\right) = V_A \cdot \left(\frac{1}{X_F} + \frac{a^2 + a}{X}\right) \tag{16}$$

Of course, $a^2 + a = -1$, and (16) becomes:

$$V_{59N} \cdot \left(\frac{X + 2 \cdot X_F}{X \cdot X_F}\right) = V_A \cdot \left(\frac{X - X_F}{X \cdot X_F}\right) \tag{17}$$

We further rearrange (17) and obtain:

$$V_{59N} = V_A \cdot \left(\frac{X - X_F}{X + 2 \cdot X_F}\right) \tag{18}$$

Switching to the per-unit frame, we write:

$$V_{59N(PU)} \cdot V_{BASE} = V_{A(PU)} \cdot V_{BASE} \cdot \left(\frac{X - X_F}{X + 2 \cdot X_F}\right) \tag{19}$$

where $V_{A(PU)} = 1\angle 0°$ (in all calculations, we assume the power system is at nominal conditions).

Therefore, (19) becomes:

$$V_{59N(PU)} = \left(\frac{X - X_F}{X + 2 \cdot X_F}\right) 1\angle 0° \tag{20}$$

Because (20) contains only the ratio of reactances, these reactances can be expressed in either ohms or per-unit values. We use Fig. 7 and calculate the healthy phase reactance, X. We use Fig. 9 (fail-open) and Fig. 10 (fail-short) to calculate the faulted-phase reactance, X_F. In per unit, we obtain:

$$X_{(PU)} = 1 \tag{21}$$

Fail-open:

$$X_{F(PU)} = R \cdot \frac{S \cdot P - F \cdot (S - 1)}{S \cdot R \cdot (P - F) + F \cdot (R - 1)} \tag{22}$$

Fail-short:

$$X_{F(PU)} = R \cdot \frac{S - F}{S \cdot R - F \cdot (R - 1)} \tag{23}$$

We insert (21) and (22) into (20) and obtain the V_{59N} operating signal for the fail-open scenario:

$$V_{59N(PU)} = \frac{F \cdot 1\angle 180°}{3 \cdot S \cdot P \cdot R - F \cdot (3 \cdot S \cdot R - 3 \cdot R + 1)} \quad (24)$$

We insert (21) and (23) into (20) and obtain the V_{59N} operating signal for the fail-short scenario:

$$V_{59N(PU)} = \frac{F \cdot 1\angle 0°}{3 \cdot S \cdot R - F \cdot (3 \cdot R - 1)} \quad (25)$$

For brevity, we omit the (PU) subscript from this point on unless necessary for clarity. The phase angles in (24) and (25) are relative to the faulted-phase voltage angle. When a capacitor unit fails open, the V_{59N} signal is out of phase with the faulted-phase voltage. When a capacitor unit fails short, the V_{59N} signal is in phase with the faulted-phase voltage. We will use this angle information when calculating the unbalance protection operating signals for multiple failures in different parts of a bank (Appendix B).

Observe the following regarding the V_{59N} signal:

- When there is no failure (F = 0), the V_{59N} operating signal is zero, as expected.
- The per-unit V_{59N} signal is a simple function of the bank parameters, S, P, and R, and does not depend on the bank ratings (nominal voltage, nominal reactive power, and nominal frequency).
- When the units fail short, the V_{59N} signal does not depend on the number of capacitor units in parallel (P).

B. Derivations

In this paper, we consider six bank configurations and several unbalance protection elements for each configuration. We consider the fail-open and fail-short failure scenarios for each bank configuration and each protection element. As a result, we derive several dozen equations to cover all the combinations.

While the unbalance equations turned out to be simple (see Appendix A), the derivation process to obtain these equations is tedious and therefore prone to errors. We have solved this challenge by using symbolic math software. Symbolic math software does not solve equations for numerical values, but instead, it manipulates equations symbolically to derive an equation for the sought variable. Symbolic math software does not output a numerical value for a numerical input. It outputs an equation based on the set of input equations. The software automates the derivation process and reduces the chance for human error.

We used the Symbolic Math Toolbox [4] in Matlab and followed this procedure:

- We wrote a set of fundamental equations for a given unbalance protection element, given bank configuration, and failure scenario.
- To avoid human errors, we did not solve or simplify these equations by hand (derivations (14) through (25) are just for illustration). Instead, we fed the original equations into the symbolic math software.
- We ran the symbolic math software to symbolically solve the set of equations for a variable of interest.
- We formatted the output equations to publish them in Appendix A in a consistent format.

C. Validation

Our derivation process is based on the symbolic math software and is therefore highly automated. Nonetheless, it is still prone to human errors related to writing the basic equations, feeding them into the software, reformatting the output equations, and typing the final equations into the text of this paper.

We have validated our equations as follows:

- We inspected each equation for expected results and symmetry with other equations. For example, when there is no failure (F = 0), all unbalance protection operating signals shall be zero.
- For each bank configuration, we ran EMTP models for several banks (banks with different parameters) and compared the values from the numerical EMTP solution to the values obtained by using equations in Appendix A.

Using the above procedure, we verified all the equations as entered in Appendix A of this paper.

V. Internal Overvoltage and Its Application in Setting the Unbalance Protection Elements

A failure in a capacitor bank causes an internal overvoltage inside the bank (see Fig. 9 and Fig. 10). This overvoltage may cause more failures, which in turn creates even higher overvoltage, and eventually, leads to a cascading failure. We propose using the overvoltage level to set trip thresholds of the unbalance protection elements. The number of failed units is only a proxy of the internal overvoltage. The same number of failed units may stress different capacitor banks differently, depending on the bank unit arrangement data and the capacitor unit voltage ratings. Tying the protection trip thresholds directly to overvoltage rather than the number of failed units is a simple and logical way to set trip thresholds.

A. Overvoltage Factor

We introduce an *overvoltage factor*, k_{OV}, as the ratio of the present voltage across a capacitor unit and the voltage across the same unit when the nominal system voltage is applied to a healthy bank. For example, when $k_{OV} = 1.15$, the voltage across the unit is 1.15 times higher than the normal voltage across the same unit. When defining the overvoltage factor, we use the normal voltage across the unit rather than the unit voltage rating to keep the voltage rating out of the equations and avoid using two per-unit voltage bases.

A capacitor unit can be safely operated when the sine wave voltage magnitude across the unit is below 110 percent of the unit nameplate voltage rating and the voltage peak value is below 120 percent [1]. Our unbalance calculations are concerned with bank failures rather than system harmonics and voltage distortion. Therefore, the 110 percent limit applies.

The nameplate voltage rating must be higher than the 1/S fraction of the system nominal phase-to-ground voltage. For example, a capacitor unit may be rated at 105 percent of the 1/S fraction of the bank nominal phase-to-ground voltage. If so, the unit can operate with an overvoltage factor, k_{OV}, of $1.05 \cdot 1.10 = 1.155$ (the voltage across the unit can increase to 115.5 percent of the normal value before the unit is in danger of failing).

We derive the overvoltage factor equations for all six bank configurations and the fail-open and fail-short scenarios and show how to use the overvoltage factor to set the trip thresholds of the unbalance protection elements.

B. Calculating the Overvoltage Factor

When F units fail open in a group (Fig. 9), the other units in that group are exposed to overvoltage. When F groups fail short in a string (Fig. 10), the other groups in the same string are exposed to overvoltage. We use the same approach as when deriving and validating equations for unbalance calculations (see Subsections IV.B and IV.C) and obtain equations for the overvoltage factor for all capacitor bank configurations and the two failure scenarios.

For example, the overvoltage factor in an ungrounded single-wye bank for the fail-open scenario is shown in (26). Also see Appendix A.

$$k_{OV} = \frac{3 \cdot S \cdot P \cdot R}{3 \cdot S \cdot P \cdot R - F \cdot (3 \cdot S \cdot R - 3 \cdot R + 1)} \tag{26}$$

Equation (26) tells us how much internal overvoltage occurs in an ungrounded single-wye bank when F units fail open. Of course, when there is no failure (F = 0), the overvoltage factor (26) equals 1, as expected.

We solve (26) to see how many units would have to fail open to cause a particular overvoltage, and we obtain:

$$F = \frac{3 \cdot S \cdot P \cdot R}{3 \cdot S \cdot R - 3 \cdot R + 1} \cdot \frac{k_{OV} - 1}{k_{OV}} \tag{27}$$

Equation (27) allows us to associate the voltage stress in the bank (k_{OV}) with the failure size (F). Consider the following example.

<u>Example 2</u>

A 138 kV ungrounded single-wye capacitor bank has its phase constructed with 14 externally fused capacitor units per group, 4 groups in series in a string, and 1 string per phase (S = 4, P = 14, and R = 1). The units are rated at 21.6 kV.

Under normal operating conditions, the voltage across the units is 138 kV / ($\sqrt{3} \cdot 4$) = 19.92 kV. The unit rating is 21.6 kV or 108.4 percent of 19.92 kV. Therefore, the overvoltage factor of safe operation is $1.1 \cdot 1.084 = 1.192$. The units can continuously withstand a 19 percent overvoltage compared with the healthy bank operated under nominal system voltage.

Assume that a single unit fails open (F = 1). Using (26), we calculate the overvoltage factor and obtain $k_{OV} = 1.063$. This value is less than 1.192 and the bank is not in danger of a cascading failure. If two units fail open (F = 2), we obtain $k_{OV} = 1.135$. When three units fail open (F = 3), we obtain $k_{OV} = 1.217$, which is above the permissible level of 1.192.

We use (27) to calculate the number of units that, if failed open, would cause the maximum permissible overvoltage of 1.192 and obtain F = 2.717 units (the bank can tolerate two failed units in the same group of 14, but not three). Ideally, the bank should be tripped when enough capacitor elements in the third capacitor unit have failed and caused the maximum permissible internal overvoltage.

Remember that (26) and (27) are examples and only apply to the fail-open scenario in ungrounded single-wye banks. Appendix A lists the overvoltage factor equations for all bank configurations and failure scenarios.

C. Using Overvoltage to Set Unbalance Protection Elements

We combine the equation that ties the operating signal of an unbalance protection element to the number of failed units:

$$\text{Operating Signal} = f(F) \tag{28}$$

with the equation that ties the number of failed units to the overvoltage factor:

$$F = g(k_{OV}) \tag{29}$$

and we obtain a direct relationship between the operating signal and the overvoltage factor:

$$\text{Operating Signal} = f\big(g(k_{OV})\big) \tag{30}$$

We use the same approach as when deriving and validating equations for unbalance calculations (see Subsections IV.B and IV.C) and derive equations for the per-unit unbalance protection operating signals as functions of the overvoltage factor. For example, we obtain (31) for the 59N element operating signal for the ungrounded single-wye bank (see Appendix A):

$$V_{59N} = \frac{k_{OV} - 1}{3 \cdot R \cdot (S - 1) + 1} \tag{31}$$

Equation (31) is very useful because it directly ties the per-unit operating signal of the 59N unbalance protection element with the internal overvoltage in the protected bank.

<u>Example 3</u>

Let us continue Example 2 and set the 59N element to trip when the internal overvoltage is at the maximum permissible level. We use 1.192 in (31) and obtain V_{59N} = 0.0193 pu or 1.93 percent of the base voltage.

The VT ratio for the 59N element in this example is 332:1. We use (11) and obtain the 59N trip threshold in secondary volts as follows:

$$V_{59N(SEC)} = 0.0193 \cdot \frac{138{,}000 \text{ V pri}}{\sqrt{3} \cdot 332} = 4.63 \text{ V sec}$$

By using the threshold of 4.63 V sec, we ensure that the 59NT/59NU protection element operates when the failure causes the internal overvoltage to exceed 110 percent of the unit voltage rating.

Fig. 11 illustrates the relationships between the 59N operating signal and the number of failed units, the overvoltage factor and the number of failed units, and the combined relationship between the 59N operating signal and the overvoltage factor for Example 2 and Example 3.

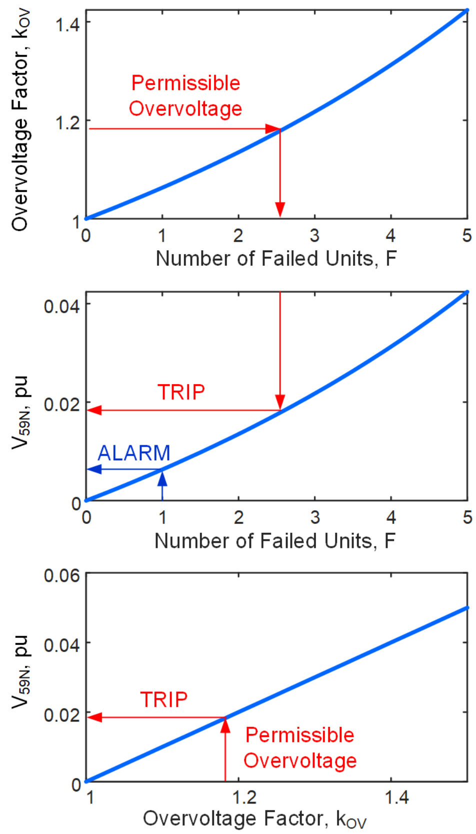


Fig. 11. Direct relationship between the 59N operating signal and the overvoltage factor (Example 2 and Example 3).

You may consider setting the 59N alarm threshold to alarm when a single unit fails (F = 1), and you may consider setting the 59N trip threshold to trip when the internal overvoltage approaches the highest permissible level (Fig. 11). A single unit failure causes V_{59N} = 0.00632 pu or 1.52 V sec (24). Therefore, you can set the 59N unbalance protection alarm threshold to 1.52 V sec and the trip threshold to 4.63 V sec.

Remember that (31) and Fig. 11 show an example and apply only to the fail-open scenario in ungrounded single-wye banks. Appendix A lists equations for all bank configurations, unbalance protection elements, and failure scenarios.

VI. Unbalance Protection Settings Calculation Examples

In this section, we illustrate the described concepts and show their benefits by providing settings calculation examples for several bank configurations and unbalance protection elements. We set the unbalance protection elements to alarm when a single unit fails and to trip when the internal overvoltage exceeds 110 percent of the unit voltage rating. For simplicity and uniformity, we do not apply setting margins or consider alarming on partial unit failures.

A. *Grounded Single-Wye Capacitor Bank*

Table I shows the bank data. We used this bank in Example 1 and explained that this nonuniform bank (different unit arrangement above and below the 87V tap) can be treated as a uniform bank with T = 0.002886 and the equivalent value of S, S = 8.023.

Table I.
Grounded Single-Wye Capacitor Bank Data

Voltage (kV LL)	230
Bus Voltage PTR	2000:1
Bank Nominal Power (MVAr)	75.14
Breaker CTR	250:5
Units in a Group, P	1
Groups in a String, S	8
Strings in a Phase, R	6
Unit Power Rating, kVAr	580
Unit Voltage Rating, kV	17.5
Unit Type	Fuseless
87V Tap	See Example 1
87V Tap PTR	3.2:1
CTs in Each String	No

The base units are as follows from (8), (9), and (10):

$$V_{BASE} = 132.79 \text{ kV}$$
$$I_{BASE} = 188.62 \text{ A}$$
$$Z_{BASE} = 704.02 \ \Omega$$

We use equations from Appendix A and apply the fail-short scenario (fuseless bank) to calculate the unbalance protection alarm and trip thresholds. We consider the per-phase 21C protection element because no CTs are installed on the per-string basis. Fig. 12 and Fig. 13 plot the unbalance protection element operating signals as functions of the number of failed units and as functions of the overvoltage factor, respectively.

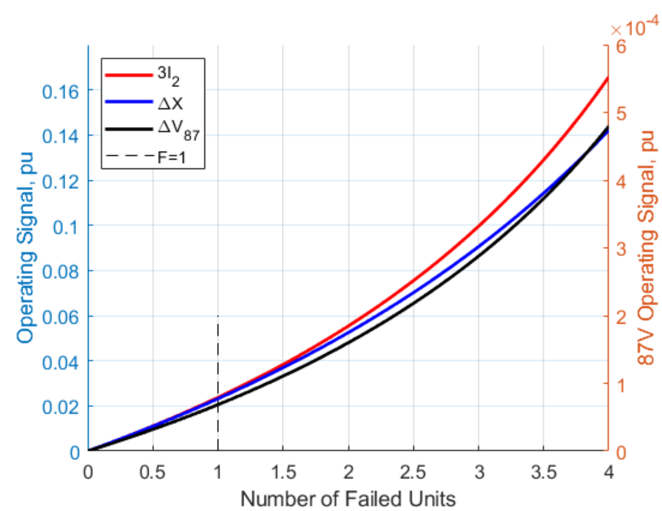


Fig. 12. Unbalance protection operating signals as functions of the number of failed units for the bank in Table I.

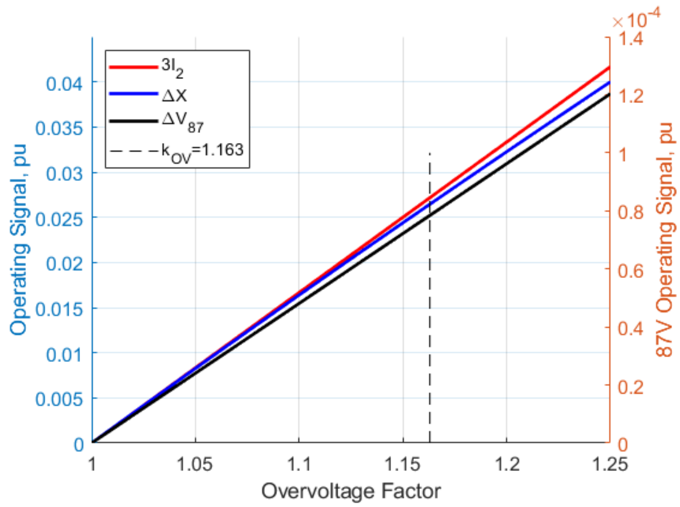


Fig. 13. Unbalance protection operating signals as functions of the overvoltage factor for the bank in Table I.

We set the unbalance protection functions to alarm when a single unit fails (F = 1) and to trip when an overvoltage due to unit failures reaches 110 percent of the unit rating of 17.5 kV. The normal voltage across each unit is 230 kV / ($\sqrt{3} \cdot 8.023$) = 16.55 kV. Therefore, we calculate the trip thresholds by using $k_{OV} = 1.1 \cdot 17.5$ kV / 16.55 kV = 1.163.

Alarm thresholds (F = 1; see Fig. 12):

$$3I_2 = 0.023732 \text{ pu}$$

$$\Delta X = -0.023181 \text{ pu}$$

$$\Delta V_{87} = 6.8513 \cdot 10^{-5} \text{ pu}$$

Trip thresholds (k_{OV} = 1.163; see Fig. 13):

$$3I_2 = 0.027167 \text{ pu}$$

$$\Delta X = -0.026448 \text{ pu}$$

$$\Delta V_{87} = 7.8434 \cdot 10^{-5} \text{ pu}$$

For illustration, let us convert the per-unit trip thresholds to secondary units by using (11), (12), and (13):

$$3I_2 = 0.027167 \text{ pu} \cdot 188.62 \text{ A} / 50 = 0.102 \text{ A sec}$$

$$\Delta X = -0.026448 \text{ pu} \cdot 704.02\ \Omega \cdot 50 / 2000 = -0.4655\ \Omega \text{ sec}$$

$$\Delta V_{87} = 7.8434 \cdot 10^{-5} \text{ pu} \cdot 132.79 \text{ kV} / 3.2 = 3.26 \text{ V sec}$$

The reactance change of –0.4655 Ω sec is a per-phase change from the nominal value of 17.601 Ω sec. The 21C element can use a blocking characteristic in the form of a circle centered at –j17.601 Ω sec and having a blocking radius of 0.4655 Ω sec.

B. Ungrounded Single-Wye Capacitor Bank

Table II shows the bank data. We used this bank in Examples 2 and 3. The base units are as follows from (8) and (9):

$$V_{BASE} = 79.674 \text{ kV}$$

$$I_{BASE} = 29.885 \text{ A}$$

We use equations from Appendix A and apply the fail-open scenario (fused bank) to calculate the unbalance protection alarm and trip thresholds. Fig. 14 and Fig. 15 plot the unbalance protection operating signals as functions of the number of failed units and as functions of the overvoltage factor, respectively.

TABLE II.
UNGROUNDED SINGLE-WYE CAPACITOR BANK DATA

Voltage (kV LL)	138
Bus Voltage PTR	1200:1
Bank Nominal Power (MVAr)	7.143
Breaker CTR	100:5
Units in a Group, P	14
Groups in a String, S	4
Strings in a Phase, R	1
Unit Voltage Rating, kV	21.6
Unit Type	Externally fused
Neutral Voltage PTR	332:1

Note that the V_{59N} and I_2 operating signals are equal in per-unit values (see Appendix A). In other words, in per-unit values, $3I_2$ and V_{59N} have a 3:1 relationship. If we plotted I_2 instead of $3I_2$, the two curves in Fig. 14 (and Fig. 15) would overlap.

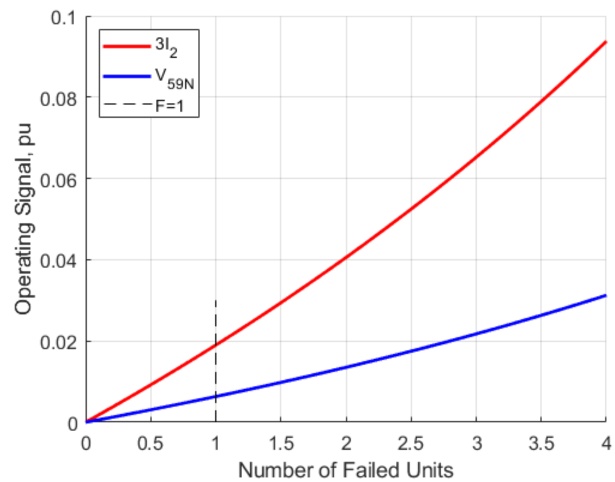


Fig. 14. Unbalance protection operating signals as functions of the number of failed units for the bank in Table II.

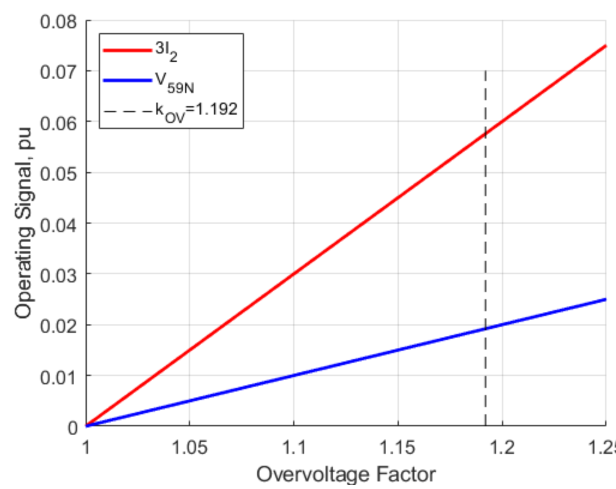


Fig. 15. Unbalance protection operating signals as functions of the overvoltage factor for the bank in Table II.

We set the unbalance protection functions to alarm when a single unit fails (F = 1) and to trip when an overvoltage due to unit failures reaches 110 percent of the unit rating of 21.6 kV.

The normal voltage across each unit is 138 kV / ($\sqrt{3} \cdot 4$) = 19.918 kV. Therefore, we calculate the trip thresholds by using k_{OV} = 1.1 · 21.6 kV / 19.918 kV = 1.192.

Alarm thresholds (F = 1; see Fig. 14):

$$3I_2 = 0.018987 \text{ pu}$$

$$V_{59N} = 0.0063291 \text{ pu}$$

Trip thresholds (k_{OV} = 1.192; see Fig. 15):

$$3I_2 = 0.0576 \text{ pu}$$

$$V_{59N} = 0.0192 \text{ pu}$$

For illustration, let us convert the per-unit trip thresholds to secondary units by using (11) and (12):

$$3I_2 = 0.0576 \text{ pu} \cdot 29.885 \text{ A} / 20 = 0.0861 \text{ A sec}$$

$$V_{59N} = 0.0192 \text{ pu} \cdot 79.674 \text{ kV} / 332 = 4.607 \text{ V sec}$$

C. *Grounded Double-Wye Capacitor Bank*

Table III shows the bank data.

TABLE III.
GROUNDED DOUBLE-WYE CAPACITOR BANK DATA

Voltage (kV LL)	138
Bus Voltage PTR	1200:1
Bank Nominal Power (MVAr)	100
Breaker CTR	2000:5
Units in a Group, P	15
Groups in a String, S	6
Strings in a Phase, R	1
Unit Voltage Rating, kV	13.8
Unit Type	Externally fused
87V Tap	One bottom group
87V Tap PTR	120:1
60N CTR	20:5

The base units are as follows from (8), (9), and (10):

$$V_{BASE} = 79.674 \text{ kV}$$

$$I_{BASE} = 418.37 \text{ A}$$

$$Z_{BASE} = 190.44\ \Omega = 63.48\ \Omega \text{ sec}$$

The bank has a uniform unit arrangement, and we calculate the tap position as follows:

$$T = 1/6 = 0.1667 \text{ pu}$$

The units can safely withstand a voltage of:

$$1.1 \cdot 13.8 \text{ kV} = 15.18 \text{ kV}$$

Normally, the units operate under a voltage of:

$$79.674 \text{ kV} / 6 = 13.279 \text{ kV}$$

Therefore, the overvoltage factor when selecting the trip threshold is:

$$k_{OV} = 15.18 \text{ kV} / 13.279 \text{ kV} = 1.143$$

We use equations from Appendix A and apply the fail-open scenario (fused bank) to calculate the unbalance protection alarm and trip thresholds. We consider the per-phase 21C protection element because the bank has only one string per phase. Fig. 16 and Fig. 17 plot the unbalance protection operating signals as functions of the number of failed units and as functions of the overvoltage factor, respectively.

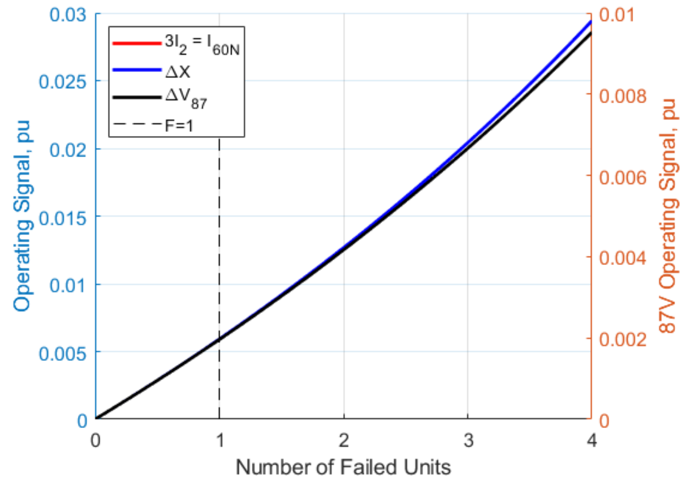


Fig. 16. Unbalance protection operating signals as functions of the number of failed units for the bank in Table III.

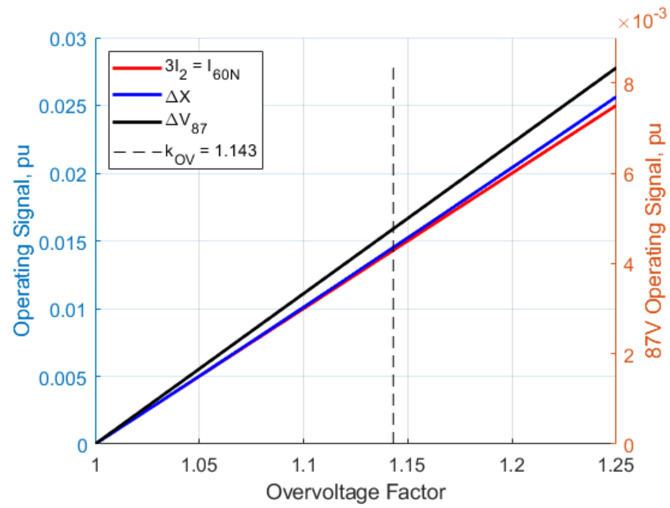


Fig. 17. Unbalance protection operating signals as functions of the overvoltage factor for the bank in Table III.

Alarm thresholds (F = 1; see Fig. 16):

$$3I_2 = I_{60N} = 0.005882 \text{ pu}$$

$$\Delta X = 0.0059172 \text{ pu}$$

$$\Delta V_{87} = 0.0019608 \text{ pu}$$

Trip thresholds (k_{OV} = 1.143; see Fig. 17):

$$3I_2 = I_{60N} = 0.0143 \text{ pu}$$

$$\Delta X = 0.014507 \text{ pu}$$

$$\Delta V_{87} = 0.0047667 \text{ pu}$$

For illustration, let us convert the per-unit trip thresholds to secondary units by using (11), (12), and (13):

$$3I_2 = 0.0143 \text{ pu} \cdot 418.37 \text{ A} / 400 = 0.0150 \text{ A sec}$$

$$I_{60N} = 0.0143 \text{ pu} \cdot 418.37 \text{ A} / 4 = 1.50 \text{ A sec}$$

$$\Delta X = 0.014507 \text{ pu} \cdot 190.44\ \Omega \cdot 400 / 1200 = 0.921\ \Omega \text{ sec}$$

$$\Delta V_{87} = 0.0047667 \text{ pu} \cdot 79.674 \text{ kV} / 120 = 3.1649 \text{ V sec}$$

The reactance change of 0.921 Ω sec is a change from the nominal value of 63.48 Ω sec.

D. *Grounded H-Bridge Capacitor Bank*

Table IV shows the bank data.

TABLE IV.
GROUNDED H-BRIDGE CAPACITOR BANK DATA

Voltage (kV LL)	345
Bus Voltage PTR	3000:1
Bank Nominal Power (MVAr)	130.89
Breaker CTR	1000:5
Units in a Group, P	1
Groups in a String, S	22
Strings in a Phase, R	2
Unit Voltage Rating, kV	9.96
Unit Type	Fuseless
Bridge Position (pu)	0.5
87V Tap PTR	1500:1
60P CTR	5:5
60N CTR	5:5

The base units are as follows from (8), (9), and (10):

$$V_{BASE} = 199.186 \text{ kV}$$

$$I_{BASE} = 219.042 \text{ A}$$

$$Z_{BASE} = 909.348\ \Omega = 60.623\ \Omega \text{ sec}$$

The units can safely withstand a voltage of:

$$1.1 \cdot 9.96 \text{ kV} = 10.956 \text{ kV}$$

Normally, the units operate under a voltage of:

$$199.186 \text{ kV} / 22 = 9.0539 \text{ kV}$$

Therefore, the overvoltage factor when selecting the trip threshold is:

$$k_{OV} = 10.956 \text{ kV} / 9.0539 \text{ kV} = 1.210$$

We use equations from Appendix A and apply the fail-short scenario (fuseless bank) to calculate the unbalance protection alarm and trip thresholds. Fig. 18 and Fig. 19 plot the unbalance protection operating signals as functions of the number of failed units and as functions of the overvoltage factor, respectively.

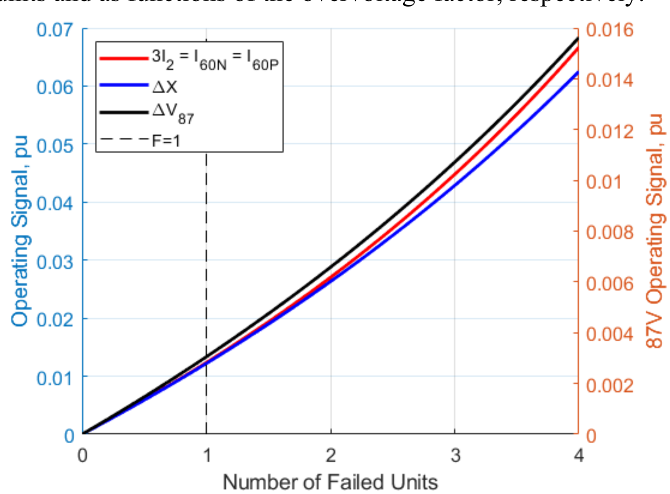


Fig. 18. Unbalance protection operating signals as functions of the number of failed units for the bank in Table IV.

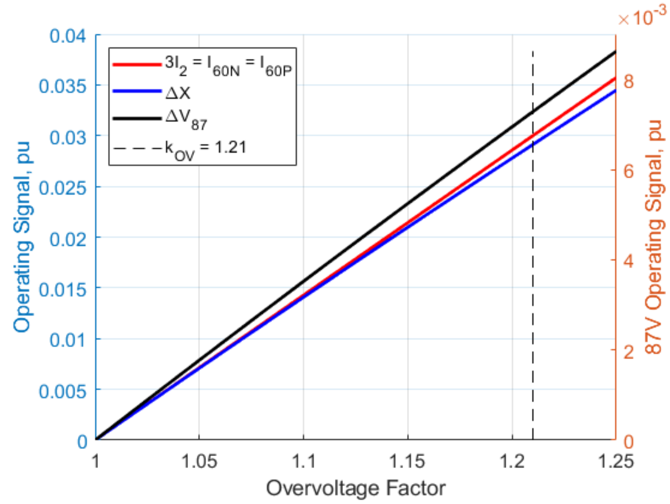


Fig. 19. Unbalance protection operating signals as functions of the overvoltage factor for the bank in Table IV.

Alarm thresholds (F = 1; see Fig. 18):

$3I_2 = I_{60N} = 0.012346$ pu

$I_{60P} = 0.012346$ pu (same as $3I_2$ because H = 0.5)

$\Delta X = -0.012195$ pu

$\Delta V_{87} = 0.0030488$ pu

Trip thresholds ($k_{OV} = 1.210$; see Fig. 19):

$3I_2 = I_{60N} = 0.03$ pu

$I_{60P} = 0.03$ pu (same as $3I_2$ because H = 0.5)

$\Delta X = -0.029126$ pu

$\Delta V_{87} = 0.0072816$ pu

For illustration, let us convert the per-unit trip thresholds to secondary units by using (11), (12), and (13):

$$3I_2 = 0.03 \text{ pu} \cdot 219.042 \text{ A} / 200 = 0.0329 \text{ A sec}$$

$$I_{60N} = 0.03 \text{ pu} \cdot 219.042 \text{ A} / 1 = 6.571 \text{ A sec}$$

$$I_{60P} = 0.03 \text{ pu} \cdot 219.042 \text{ A} / 1 = 6.571 \text{ A sec}$$

$$\Delta X = 0.029126 \text{ pu} \cdot 909.348\ \Omega \cdot 200 / 3000 = 1.766\ \Omega \text{ sec}$$

$$\Delta V_{87} = 0.0072816 \text{ pu} \cdot 199.186 \text{ kV} / 1500 = 0.9669 \text{ V sec}$$

VII. INSIGHTS INTO THE UNBALANCE PROTECTION ELEMENTS

In this section, we provide insights into the unbalance protection elements based on the equations in Appendix A.

A. *Some Unbalance Protection Element Operating Signals Are Identical or Proportional to One Another*

For a series failure in one location, the per-unit operating signals of some unbalance protection elements are identical or proportional to one another. As a result, these protection elements can be considered as redundant elements rather than complementary elements that mutually cover their weak spots.

Consider the following three examples:

Any ungrounded bank (per-unit values):

$$V_{59N} = -j\frac{1}{3} \cdot 3I_2 \tag{32}$$

Grounded double-wye bank:

$$I_{60P} = I_{60N} = 3I_2 \tag{33}$$

Ungrounded double-wye bank:

$$I_{60P} = 3I_2, \qquad I_{60N} = \frac{1}{2} \cdot 3I_2 \tag{34}$$

We draw the following insights and observations regarding the unbalance protection elements that have the same or proportional per-unit operating signals.

Even though these elements have the same theoretical sensitivity, their practical applications differ. For example, CT ratio and angle errors reduce the accuracy of measuring the $3I_2$ operating signal. A capacitor bank protective relay may see a standing $3I_2$ signal for a healthy bank because of small CT errors. By contrast, there will be no standing V_{59N}, I_{60N}, and I_{60P} signals (assuming that window CTs are used to measure the unbalance currents). As a result, the 59NT/59NU, 60N, and 60P protection elements can be set with more sensitivity than the 50QT element.

These elements have the same theorical sensitivity, but only for series failures in a single location. Their sensitivities differ for shunt failures, such as phase-to-ground and phase-to-phase faults, and for multiple series failures at different locations (see Appendix B). As a result, enabling unbalance protection elements that have identical or proportional operating signals is still justified. These elements are redundant for a single capacitor unit failure but complementary for multiple capacitor unit failures and for phase-to-ground and phase-to-phase faults.

B. *All Unbalance Protection Element Operating Signals Are Similar*

As an example, consider the grounded single-wye bank configuration under the fail-open scenario and examine the $3I_2$ operating signal and the per-phase ΔX operating signal:

$$|3I_2| = \frac{F}{S \cdot R \cdot P - F \cdot (S \cdot R - R)} \tag{35}$$

$$|\Delta X| = \frac{F}{S \cdot R \cdot P - F \cdot (S \cdot R - R + 1)} \tag{36}$$

The two equations are almost identical and differ only by 1 in the multiplier following the number of failed units, F, in the denominator (shown in red). The number of series groups in a string, S, is much greater than 1. Therefore, the values of $R \cdot (S - 1)$ and $R \cdot (S - 1) + 1$ are almost identical, and we conclude that in per unit:

$$|\Delta X| \cong |3I_2| \tag{37}$$

In other words, the per-phase 21C element and the 50QT element have almost identical theoretical sensitivities to capacitor unit failures. The 21C element uses one voltage transformer and one current transformer per phase and the 50QT element uses three current transformers. Therefore, the operating signals of the 21C and 50QT elements may experience different measurement errors. Based on these errors, it may be possible to apply more sensitive settings to the 21C element or the 50QT element. Of course, using the per-string rather than per-phase 21C protection element would dramatically improve the 21C sensitivity and favor it over the 50QT element (see Subsection VII.G).

In general, the product of the P and R parameters is significantly higher than 1 (for survivability and to obtain the required power rating, we have multiple parallel units in a group or parallel strings in a phase, or both). Therefore, all unbalance protection elements (for a given bank configuration and for a given failure scenario) have operating signals that are either identical, proportional to one another, or very similar. Of course, the elements' settings in secondary units differ, but this is because of the physical nature of the operating signals (voltage, current, and reactance), instrument transformer ratios, and tap-matching.

C. *All Unbalance Equations Have a Common Format*

The unbalance equations display a significant similarity across all bank configurations and protection elements. Consider the following examples:

Ungrounded single-wye bank, fail-open:

$$|V_{59N}| = \frac{F}{3 \cdot S \cdot P \cdot R - F \cdot (3 \cdot S \cdot R - 3 \cdot R + 1)} \tag{38}$$

Grounded double-wye bank, fail-short:

$$|3I_2| = \frac{1}{2} \cdot \frac{F}{S \cdot R - F \cdot R} \tag{39}$$

Grounded H-bridge, fail-open (G = 1 – H for failures above the bridge, and G = H for failures below the bridge):

$$|\Delta V_{87}| = \frac{H \cdot G \cdot F}{4 \cdot S \cdot R \cdot P \cdot G - F \cdot (4 \cdot S \cdot R \cdot G - 4 \cdot R + 2)} \tag{40}$$

All these equations have the same general format:

$$Y = \frac{k \cdot F}{A - B \cdot F} \tag{41}$$

where factors k, A, and B are positive numbers that are specific to each protection element and depend on the capacitor unit arrangement and bank configuration, and Y stands for a general unbalance protection element operating signal.

Equation (41) teaches us that when the failure size is small, the unbalance protection operating signals are near-linear functions of the number of failed elements, F. When the failure size increases, the denominator in (41) decreases and the operating signal (41) becomes a steeper function of the failure size (see Fig. 12, Fig. 14, Fig. 16, and Fig. 18).

We can use (41) to evaluate sensitivity of the unbalance protection elements across bank configurations, especially for small failures (alarm application). We want to know the increase in the operating signal (ΔY) for an increase in the failure size (ΔF). We use the derivative of (41) and write:

$$\Delta Y \cong \frac{d}{dF}\left(\frac{k \cdot F}{A - B \cdot F}\right) \cdot \Delta F \tag{42}$$

The slope in (42) tells us how sensitive a given protection element is. We calculate the slope from (41) and obtain:

$$\frac{d}{dF}\left(\frac{k \cdot F}{A - B \cdot F}\right) = \frac{k \cdot A}{(A - B \cdot F)^2} \quad (43)$$

When the bank is healthy, then F = 0. Therefore, when the failure develops (F increases from 0 to 1), the slope is:

$$\frac{d}{dF}\left(\frac{k \cdot F}{A - B \cdot F}\right)_{F=0} = \frac{k}{A} \quad (44)$$

When more capacitor units fail, the numerator in (43) decreases and the slope increases (sensitivity increases). Therefore, (44) is a conservative estimate of the slope and therefore sensitivity. The unbalance protection sensitivity is proportional to k and inversely proportional to A.

To illustrate this point, let us use the grounded H-bridge bank with the fail-open scenario and consider the 50QT and 87V elements. From Appendix A, the 50QT element operating signal has an initial slope of G/(2SPRG) or 1/(2SPR). The 87V element operating signal has the initial slope of HG/(2SPRG) or H/(2SPR). The two elements differ in slope only by the fixed multiplier of H. The VT ratio for the tap voltage effectively eliminates this fixed difference, and the two protection elements have effectively identical sensitivities to capacitor unit failures in a single location. That sensitivity is inversely proportional to the product of the bank unit arrangement parameters S, R, and P.

From Appendix A, we see that for all bank configurations, the k/A factor in (44) is inversely proportional to the product of S, P, and R in the fail-open scenario and inversely proportional to the product of S and R in the fail-short scenario. In other words, for a given bank configuration and fusing method, all applicable unbalance protection elements have approximately the same theoretical sensitivity to the number of failed capacitor units when considered in per unit of the bank nominal values. The practical sensitivities differ because of different instrument transformer errors and ratios.

When we consider the unbalance protection element operating signals as functions of the internal overvoltage caused by the failure (trip application), we conclude that the common equation format is as follows:

$$Y \cong C \cdot (k_{OV} - 1) \quad (45)$$

where the multiplier C is specific to each protection element, depends on the bank unit arrangement parameters, and depends slightly on the value of the overvoltage factor, k_{OV} (strictly speaking, C is not a constant).

Equation (45) teaches us that the relationship between the unbalance protection operating signals and the level of internal overvoltage caused by the failure is near linear. For example, if the overvoltage doubles, such as when k_{OV} increases from 1.10 to 1.20 (0.2 = 2 · 0.1), the operating signal doubles as well (approximately). Adjusting an unbalance protection trip threshold up or down proportionally increases or decreases the additional voltage that the failure puts on the healthy capacitor units before the element trips the bank offline or a cascading failure occurs.

From Appendix A, we see that for all bank configurations, factor C in (45) is inversely proportional to the product of S and R in the fail-open scenario and inversely proportional to R in the fail-short scenario. In other words, for a given bank configuration and fusing method, all applicable unbalance protection elements have approximately the same theoretical relationship to the level of internal overvoltage when considered in per unit of the bank nominal values.

D. Approximation of the Unbalance

From the previous subsection, we know that the unbalance protection operating signals are proportional to the following expressions:

Fail-open:

$$\Delta Y \approx \frac{1}{S \cdot P \cdot R} \cdot \Delta F \quad (46)$$

Fail-short:

$$\Delta Y \approx \frac{1}{S \cdot R} \cdot \Delta F \quad (47)$$

The number of groups in series, S, and the capacitor unit rated voltage, V_U, must be selected to satisfy the following condition (for simplicity, we neglect the margin for sustained system overvoltages):

$$V_U \geq \frac{V_{NOM}}{\sqrt{3} \cdot S} \quad (48)$$

The rated bank power is:

$$Q_{NOM} = \frac{{V_{NOM}}^2}{X} \quad (49)$$

We use (5) for the phase reactance X and use (48) for the nominal voltage and rewrite (49) as follows:

$$Q_{NOM} = 3 \cdot S \cdot P \cdot R \cdot \frac{{V_U}^2}{X_U} \quad (50)$$

The squared unit voltage rating divided by the unit reactance is the unit rated power (Q_U), therefore, we can write:

$$S \cdot P \cdot R = \frac{Q_{NOM}}{3 \cdot Q_U} \quad (51)$$

We insert (51) into (46) and (47) and obtain the unbalance protection sensitivity estimates by using the bank nominal voltage and power and the capacitor unit rated voltage and power:

Fail-open:

$$\Delta Y \approx \left(3 \cdot \frac{Q_U}{Q_{NOM}}\right) \cdot \Delta F \quad (52)$$

Fail-short:

$$\Delta Y \approx \left(3 \cdot \frac{Q_U}{Q_{NOM}} \cdot P\right) \cdot \Delta F \quad (53)$$

The product of Q_U and P is the rated power of a capacitor group. The product of the number of failed units (ΔF) and the unit power or group power is the power lost due to the failure. Keeping these observations in mind, we can consolidate (52) and (53) into a general approximation of the unbalance protection operating signal as follows:

$$\Delta Y \approx 3 \cdot \frac{\Delta Q_F}{Q_{NOM}} \approx \frac{\text{Number of Lost Units}}{\text{Number of Units per Phase}} \tag{54}$$

where ΔQ_F is the reactive power lost because of the failure (fused units became an open circuit and unfused units shorted the entire group).

You can use (54) to obtain a very quick estimate of the unbalance protection operating signals (in per unit) for a small number of failed units based on the simple count of units lost relative to the total number of units in a phase.

The value of ΔY in (54) refers to the $3I_2$ current. Calculate other unbalance quantities by factoring in the proportion between $3I_2$ and other signals (see Appendix A, Subsection VII.A and Subsection VII.B).

<u>Example 4</u>

Let us consider the grounded double-wye bank from Subsection VI.C and use (54) to get the first approximation of the per-unit unbalance. The bank has 15 capacitor units in a group, 6 groups in a string, and 1 string per phase. The bank is a double bank with two phases in parallel. The total number of units per phase is $15 \cdot 6 \cdot 1 \cdot 2 = 180$. The bank is externally fused, and therefore, a single unit failure removes one unit from the bank (a shorted unit would remove the entire group, i.e., 15 units). Using (54), we calculate the per-unit unbalance for F = 1 as 1 / 180 = 0.00555 pu.

For comparison, the exact values we calculated were: $3I_2 = I_{60N} = 0.005882$ pu and $\Delta X = 0.0059172$ pu. The approximation of (54) provided an accuracy with better than a 6 percent error.

E. Optimizing Unit Arrangement for More Sensitive Unbalance Protection

Can we select the S, R, and P parameters to increase the protection sensitivity? Equation (54) teaches us that the sensitivity to capacitor unit failures depends on the number of units lost due to the failure and the total number of units in the bank. When F units fail open in a fused bank, the number of lost units is F. When F units fail short in a fuseless bank, the number of lost units is $F \cdot P$ (F groups are lost).

The above observation applied to the approximation of (54) leads to the following conclusions:

- Sensitivity of unbalance protection for fused capacitor banks does not depend on how the capacitor units are divided between the groups and strings.
- Sensitivity of fuseless banks increases when more capacitor units are placed in groups while the number of strings is reduced.

<u>Example 5</u>

Let us use the fuseless, grounded single-wye bank from Subsection VI.A and recalculate the $3I_2$ operating signal (as an example) for different combinations of P and R while keeping the product of the two parameters at 6, as in the original bank data. Table V shows the results assuming two failures: a failure of a single unit (alarm) and a larger failure that leads to the maximum permissible internal overvoltage (trip).

TABLE V.
$3I_2$ VALUES FOR DIFFERENT P-R COMBINATIONS

P	R	$3I_2$ (pu)	
		F = 1 (alarm)	**k_{OV} = 1.163 (trip)**
1	6	0.0237	0.0272
2	3	0.0475	0.0543
3	2	0.0712	0.0815
6	1	0.1424	0.1630

Table V shows a clear increase in the operating signal when the capacitor units are moved from strings to groups. This tradeoff between P and R can be explained as follows. The parallel strings (larger R) obfuscate the unbalance caused by the failure in the faulted string and make it more difficult to detect. At the same time, shorting a larger group of units (larger P) increases the unbalance and makes it easier to detect.

Of course, considerations other than unbalance protection sensitivity apply when making a tradeoff between the number of units in a group and the number of strings in a phase [5].

Let us now look at unbalance protection sensitivity as a function of internal overvoltage. Using (45) and obtaining factor C from Appendix A, we write the following approximations:

Fail-open:

$$\Delta Y \approx \frac{1}{S \cdot R} \cdot \Delta k_{OV} \tag{55}$$

Fail-short:

$$\Delta Y \approx \frac{1}{R} \cdot \Delta k_{OV} \tag{56}$$

Equations (55) and (56) show that the unbalance protection sensitivity as a function of internal overvoltage does not depend on P and increases when R decreases. Therefore, for any given product of P and R required to obtain the target rated power given the value of S, it is advantageous to select a smaller R. This means that it is better for protection sensitivity to have more units in a group than more strings in a phase (see Example 5). For example, if the preferred product of P and R is 6, it is better to have 3 units in a group (P = 3) and 2 strings in a phase (R = 2) than the other way around.

F. Optimizing the Tap and Bridge Position for More Sensitive Unbalance Protection

Another consideration related to optimizing protection sensitivity is the placement of the 87V tap (T) or bridge (H). To evaluate the impact of T and H on protection sensitivity, we can use equations from Appendix A, consider the bank unit arrangement parameters and the failure size as constants, and treat T (or H) as a variable. We can then determine the value of T (or H) that maximizes the unbalance protection operating signal for failures above and below the tap or bridge point.

Consider the grounded H-bridge bank configuration. The unbalance protection operating signals are proportional to G. G = 1 – H for failures above the bridge and G = H for failures below the bridge. We want to maximize the smaller of the two operating signals (the signal for failures above and below the bridge). Fig. 20 plots the (1 – H) expression (the multiplier for failures above the bridge) and the H expression (the multiplier for failures below the bridge). When H is small (below 0.5), the sensitivity for failures below the bridge is lower than for failures above the bridge. This is because when located lower, the bridge is better able to short the units located below the bridge and by doing so, obfuscates failures below the bridge. When H is large (above 0.5), the sensitivity for failures above the bridge is lower than for failures below the bridge. This is because when located higher, the bridge is better able to short the units located above the bridge and by doing so, obfuscates failures above the bridge. We obtain the best sensitivity when the bridge is in the middle (H = 0.5), as is typically the case. For H = 0.5, the sensitivity is equal for failures above and below the bridge.

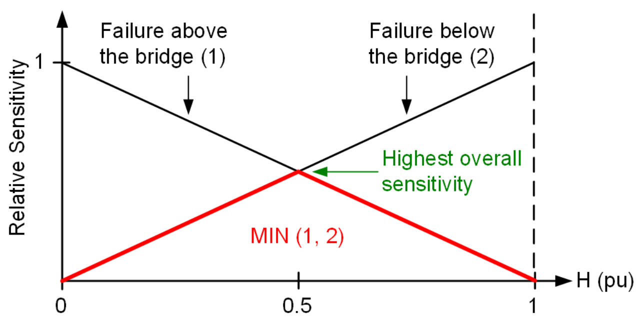


Fig. 20. Relative sensitivity for failures above and below the bridge as a function of bridge position for the H-bridge bank configuration.

We obtain a similar result for the position of the tap (T) and the 87V protection element. When T is small (tap located low), the 87V element has a much higher sensitivity to failures below the tap than for failures above the tap. When T is large (tap located high), the 87V element has a much higher sensitivity to failures above the tap than for failures below the tap. The two sensitivities are equal, and therefore, the overall 87V sensitivity is at a maximum when the tap is in the middle (T = 0.5). A higher T value increases the voltage level at the tap and requires using VTs of a higher voltage rating. Therefore, the value of T is often kept low (the tap is installed low, and the VT ratio is selected low to boost the secondary voltage signal). However, the 87V sensitivity can be improved by locating the tap closer to the midpoint.

G. Per-String vs. Per-Phase Impedance Protection

Appendix A lists the per-phase and per-string reactance changes for the grounded single- and double-wye banks. We do not derive the per-string reactance change for the grounded H-bank because the bridge connects the strings in the left and right halves of the bank and reduces the sensitivity of the per-string reactance measurement. In ohms, the per-string reactance change is the same for the single- and double-wye bank configurations. In the per-unit frame, these reactance changes differ by a factor of 2 because the base reactance is twice as low in the double-wye bank as in the single-wye bank.

Consider the grounded single-wye bank and the fail-open scenario. The sensitivities of the per-phase and per-string reactance changes are as follows:

$$\Delta X_{PHASE} \approx \frac{1}{S \cdot P \cdot R} \cdot \Delta F \tag{57}$$

$$\Delta X_{STRING} \approx \frac{R}{S \cdot P} \cdot \Delta F \tag{58}$$

Comparing the two sensitivities (slopes in (57) and (58)), we conclude that the per-string reactance change is R^2-fold higher than the per-phase reactance change. The R^2-fold difference is in per unit of the bank reactance or in secondary ohms. As a percentage of the string reactance, the per-string reactance change is R-fold higher than the percentage change of the phase reactance.

H. Accounting for Bus Voltage Fluctuations

Because a capacitor bank is a linear circuit, the unbalance protection operating signals are directly proportional to the terminal (bus) voltage. Our equations provide the unbalance protection operating signals in per unit. You can use a simple multiplier to account for changes in the terminal voltage.

To illustrate this point, let us consider the alarm and trip protection applications.

In the alarm application, the intent is to detect a single unit failure (or a partial unit failure). The operating signal during the failure decreases if the terminal voltage decreases. Therefore, it is good practice to set the alarm threshold at 0.8 times the calculated value to account for a possible 20 percent reduction in the terminal voltage during stressed system conditions. This margin allows dependable pickup of the alarm function, and it prevents deassertion of the alarm (if not latched) when the voltage decreases after the alarm is already set.

In the trip application, the intent is to trip the capacitor bank before the internal overvoltage caused by the failure breaches the unit voltage rating. From this perspective, the system overvoltage (the terminal voltage is higher than nominal) and the internal overvoltage (there is a failure in the bank that distributes the voltage unequally among the healthy capacitor units) compound. However, because the unbalance protection operating signals are proportional to the terminal voltage, there is no need for additional margin to account for terminal voltage fluctuations. Any given failure (number of failed units, F) results in an internal overvoltage that is proportional to the terminal voltage and therefore the system overvoltage. A failure at a time when the voltage is nominal may result in an internal overvoltage that is permissible and the unbalance protection may not trip, but the same failure at a time when the system voltage is higher than nominal may result in an internal overvoltage that is not permissible, and the unbalance protection may trip.

Example 6

Let us continue the ungrounded single-wye bank example from Subsection VI.B. We consider a failure of two units ($F = 2$) and inspect the 59NT/59NU element operating signal during normal voltage and during a system overvoltage. The element is set at 4.607 V sec with the intent to trip the bank when the internal voltage reaches the maximum permissible value of 110 percent of the unit voltage rating ($k_{OV} = 1.192$).

Under nominal voltage, the failure of two units results in a V_{59N} operating signal of:

$$V_{59N} = 0.013514 \text{ pu} = 3.243 \text{ V sec}$$

This operating signal is below the tripping threshold of 4.607 V sec and the 59NT/59NU element restrains. Under these conditions (two units failed under nominal system voltage), the overvoltage factor is 1.135, which is below the maximum permissible value of 1.192.

Assume first that after the failure, the capacitor bank is exposed to a 10 percent system overvoltage (terminal voltage is 1.1 times nominal). Now, the 59N operating signal is proportionally higher because of the compounding of the failure and the higher terminal voltage:

$$V_{59N} = 1.1 \cdot 3.243 \text{ V sec} = 3.567 \text{ V sec}$$

The 59N operating signal is still below the tripping threshold of 4.607 V sec and the 59NT/59NU element does not operate. Under these conditions (two units failed and the system voltage increased to 110 percent of nominal), the overvoltage factor is $1 + 1.1 \cdot (1.135 - 1) = 1.149$. This value is below the maximum permissible value of 1.192, and therefore, the 59NT/59NU element restrains as desired. The system voltage multiplier (1.1) applies to the fractional value of the k_{OV} factor because the unbalance protection operating signals are proportional to $k_{OV} - 1$ not k_{OV}. It would take a 42 percent overvoltage ($4.607 / 3.243 = 1.42$) to make the 59NT/59NU element operate. The smaller the failure size, the lower the sensitivity of the unbalance protection elements to system overvoltage.

Without a failure (or under equalizing failures), the unbalance protection elements do not detect the system overvoltage at all. They detect system overvoltage only when the failure size causes an operating signal that is close to the trip threshold under nominal voltage. Therefore, unbalance protection does not substitute for bank overvoltage protection, and you need to apply phase overvoltage elements (59P) to protect against system overvoltages without and with harmonics [1] [5]. At the same time, no additional margin is needed when setting the unbalance protection trip thresholds to account for overvoltage because the increased terminal voltage changes the internal overvoltage and the unbalance protection operating signals proportionally.

The 21C element operates irrespective of the terminal voltage, which is a slight disadvantage when considering voltage fluctuations. As a result, the 21C element shall be set with an additional margin to account for system overvoltages. For example, if the maximum expected overvoltage is 15 percent, you may set the trip threshold for the ΔX value at 1/1.15 (or 0.87) times the threshold calculated with the Appendix A equations. This is because when the terminal voltage is elevated, it takes a smaller impedance unbalance to cause the same internal overvoltage in the bank.

VIII. Conclusions

In this paper, we derived equations for unbalance calculations for six common high-voltage capacitor bank configurations. Expressed in per unit, the unbalance protection operating signals are simple functions of the bank unit arrangement parameters: number of units in a group, number of groups in a string, and number of strings in a phase. The per-unit unbalance protection operating signals are independent of many other factors, such as unit reactance and power rating, frequency, and bank nominal voltage and power.

For completeness, our equations cover both the fail-open and fail-short failure scenarios for all bank configurations. Not all fusing methods are applied to all bank configurations. However, to calculate unbalance under pre-existing failures and temporary bank repairs, we need both the fail-open and fail-short scenarios for all bank configurations regardless of the fusing method.

These equations allow analyzing bank failures as well as setting the unbalance protection elements to alarm and trip.

In the context of capacitor bank analysis, the equations allow unbalance calculations in a quick and convenient way. Our equations apply to a single failure. You can use the principle of superposition to perform calculations for multiple failures occurring sequentially in different parts of the bank (Appendix B). Following this approach, you can also analyze the impact of inherent bank unbalance due to past failures or temporary repairs.

We introduced the concept of an overvoltage factor: the ratio of the voltage across a capacitor unit elevated because of the failure and the normal voltage across that unit. The paper derives overvoltage factor equations for the common bank configurations under both the fail-open and fail-short failure scenarios. These equations allow evaluating the voltage stress that a failure puts on the healthy capacitor units in the bank.

In the context of setting the bank protection elements, we propose to set the unbalance protection alarm thresholds to detect a single (or partial) capacitor unit failure. We propose to set the unbalance protection trip thresholds by using the overvoltage factor. The trip is issued when the overvoltage caused by the failure reaches the voltage rating of the units in the bank. The provided equations allow calculating the trip thresholds directly from the highest permissible overvoltage factor given the bank unit arrangement parameters.

We examined the derived equations and gathered the following important conclusions:

- The unbalance equations are similar to one another in terms of both the expression and the values they return for typical bank unit arrangement data.

- There is a near-linear relationship between the unbalance protection operating signals and the overvoltage level caused by the failure.
- There is a near-linear relationship between the unbalance protection operating signals and the failure size.
- For small failures, the unbalance protection operating signals for all methods are almost identical or proportional to one another and depend on the amount of reactive power lost because of the failure relative to the nominal power in one phase.

We also showed (Appendix C) how to perform unbalance calculations for capacitor element failures (partial unit failures), including the following cases:

- Some but not all fuses have blown in internally fused banks.
- Some but not all capacitor elements have shorted in the fuseless banks and externally fused banks before the external (unit) fuse blows.

Our method for calculating the unbalance for capacitor element failures is very simple because it decouples the bank arrangement (units in groups, groups in strings, strings in phases, phase connections, and grounding) from the arrangement of capacitor elements inside capacitor units.

The paper uses many numerical examples to explain and illustrate the content. Appendix A is a compilation of all the unbalance equations that you can print and use when performing protection calculations for high-voltage capacitor banks.

IX. Appendix A. Capacitor Bank Unbalance Calculation Equations

This appendix lists equations for unbalance calculations for the six capacitor bank configurations in six separate tables. Each table includes equations for the fail-open and fail-short scenarios and shows the bank configuration, measurements, and unbalance protection elements marked with the ANSI device numbers. The unbalance equations are in per unit. When appropriate, the equations include phase angles that you can use to apply the principle of superposition when performing calculations for simultaneous failures at two or more locations.

The equations use the following symbols:

- P Number of capacitor units in parallel in a group
- S Number of groups in series in a string
- R Number of strings in parallel in a phase
- F Number of failed capacitor units (see Fig. 9 and Fig. 10 for the fail-open and fail-short scenarios, respectively)
- T Per-unit tap for the 87V element (per-unit reactance of the bottom part)
- H Per-unit position of the bridge in an H-bridge bank
- k_{OV} Overvoltage factor (the ratio of the voltage across a healthy capacitor unit during a failure and during nominal conditions, $k_{OV} = 1$ when there is no failure)

Remember that the overvoltage factor, k_{OV}, refers to the normal voltage, and not the unit rating (V_U). The unit rating is typically higher than the normal voltage and the units are designed to withstand 110 percent of the rated value. Typically, when calculating a trip threshold, you will use k_{OV} of 1.1 times the unit voltage rating divided by the unit voltage during nominal system conditions:

$$k_{OV} = 1.1 \cdot \frac{V_U}{\left(\frac{V_{NOM}}{\sqrt{3} \cdot S}\right)} \tag{A.1}$$

where V_{NOM} is the nominal (phase-to-phase) bank voltage.

Before using the equations, ensure that the capacitor unit arrangement in your capacitor bank conforms to the assumptions of this paper (see Fig. 7).

When performing unbalance calculations for capacitor element failures (partial capacitor unit failures), calculate and use the fractional failure size as follows (see Appendix C):

Fail-Open	Fail-Short	
$F_{FRAC} = 1 - \alpha$	$F_{FRAC} = \dfrac{\alpha - 1}{\alpha - 1 + P}$	(A.2)

where:

$$\alpha = \frac{X_U}{X_{UF}} \tag{A.3}$$

X_U is the reactance of a healthy capacitor unit

X_{UF} is the reactance of a partially failed capacitor unit

Refer to the body of the paper for more details about the following topics:

- Tap-matching of the 59NU and 87V operating signals (Section II).
- Equivalencing banks with nonuniform unit arrangement above and below the 87V tap point (Section III.A).
- Base values for the per-unit voltage, current, and reactance and conversion to secondary values (Section III.C).

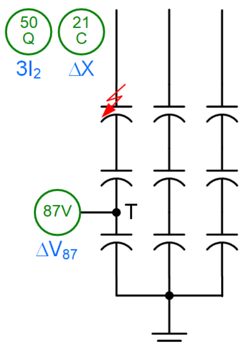
Following are typical use cases for the material in this appendix:

- Assume the size and location of a unit failure (F) and calculate the unbalance protection operating signals.
- Assume the size and location of multiple unit failures and calculate the unbalance protection operating signals by using the principle of superposition.
- Assume the size and location of a unit failure (F) and calculate the alarm threshold for the unbalance protection elements.
- Assume the size and location of a unit failure (F) and calculate the overvoltage factor for the healthy units (k_{OV}) because of the failure.
- Assume the maximum permissible overvoltage factor (k_{OV}) and calculate the number of failed units (F) that would cause that level of overvoltage.
- Assume the maximum permissible overvoltage factor (k_{OV}) and calculate the trip threshold for the unbalance protection elements.
- Use the angle of the operating signals in the post-fault analysis to identify the faulted phase and location (above or below the tap/bridge, left or right half of the bank, and the faulted phase).

To keep the equations in a single table on one page, the equations omit the multiplication sign (·). For example, because there is no variable called SP, SP is a product of the two variables (S · P).

Grounded Single-Wye Capacitor Bank

Fail-Open	Fail-Short
$k_{OV} = \dfrac{SP}{SP - F(S-1)}$ $F = \dfrac{SP}{S-1}\dfrac{k_{OV}-1}{k_{OV}}$	$k_{OV} = \dfrac{S}{S-F}$ $F = S\dfrac{k_{OV}-1}{k_{OV}}$
$3I_2 = \dfrac{F}{SRP - F(SR-R)} 1\angle{-90°}$ $3I_2 = \dfrac{k_{OV}-1}{R(S-1)} 1\angle{-90°}$	$3I_2 = \dfrac{F}{SR - FR} 1\angle 90°$ $3I_2 = \dfrac{k_{OV}-1}{R} 1\angle 90°$
$\Delta X_{PHASE} = \dfrac{F}{SRP - F(SR - R + 1)}$ $\Delta X_{PHASE} = \dfrac{k_{OV}-1}{R(S-1)+1-k_{OV}}$	$\Delta X_{PHASE} = -\dfrac{F}{SR - F(R-1)}$ $\Delta X_{PHASE} = -\dfrac{k_{OV}-1}{R + k_{OV} - 1}$
$\Delta X_{STRING} = R\dfrac{F}{SP - FS}$ $\Delta X_{STRING} = R\dfrac{k_{OV}-1}{S-k_{OV}}$	$\Delta X_{STRING} = -\dfrac{R}{S}F$ $\Delta X_{STRING} = -R\dfrac{k_{OV}-1}{k_{OV}}$
$\Delta V_{87} = \dfrac{TF}{SRP - F\left(SR - \dfrac{R-T}{1-T}\right)} 1\angle 0°$ $\Delta V_{87} = \dfrac{T(1-T)(k_{OV}-1)}{R(S-1) - T(SR - (R-1)k_{OV} - 1)} 1\angle 0°$	$\Delta V_{87} = \dfrac{TF}{SR - F\dfrac{R-T}{1-T}} 1\angle 180°$ $\Delta V_{87} = \dfrac{T(1-T)(k_{OV}-1)}{R - T\big((R-1)k_{OV}+1\big)} 1\angle 180°$



Notes:

All values are in per unit.

ΔX_{PHASE} and ΔX_{STRING} are both in per unit of the bank reactance.

Voltage and current phase angles are relative to the faulted-phase voltage.

Δ87V differential signal uses bus voltage scaled down to the tap voltage ($V_{TAP} - T \cdot V_{BUS}$).

$$T = \frac{X_{BOTTOM}}{X_{TOP} + X_{BOTTOM}}$$

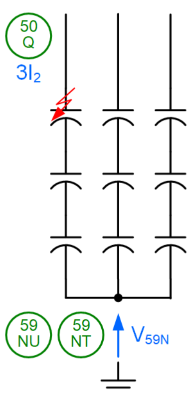
For nonhomogeneous banks (different unit arrangement above and below the tap), P and R are parameters of the top part and S is an equivalent value, as follows:

$$S = \frac{S_{TOP}}{1 - T}$$

Ungrounded Single-Wye Capacitor Bank

Fail-Open	Fail-Short
$k_{OV} = \dfrac{3SPR}{3SPR - F(3SR - 3R + 1)}$ $F = \dfrac{3SPR}{3SR - 3R + 1} \dfrac{k_{OV} - 1}{k_{OV}}$	$k_{OV} = \dfrac{3SR}{3SR - F(3R - 1)}$ $F = \dfrac{3SR}{3R - 1} \dfrac{k_{OV} - 1}{k_{OV}}$
$3I_2 = \dfrac{3F}{3SPR - F(3SR - 3R + 1)} 1\angle{-90°}$ $3I_2 = 3 \dfrac{k_{OV} - 1}{3R(S - 1) + 1} 1\angle{-90°}$	$3I_2 = \dfrac{3F}{3SR - F(3R - 1)} 1\angle 90°$ $3I_2 = 3 \dfrac{k_{OV} - 1}{3R - 1} 1\angle 90°$
$V_{59N} = \dfrac{F}{3SPR - F(3SR - 3R + 1)} 1\angle 180°$ $V_{59N} = \dfrac{k_{OV} - 1}{3R(S - 1) + 1} 1\angle 180°$	$V_{59N} = \dfrac{F}{3SR - F(3R - 1)} 1\angle 0°$ $V_{59N} = \dfrac{k_{OV} - 1}{3R - 1} 1\angle 0°$

$$V_{59N} = -j\frac{1}{3}3I_2$$



Notes:

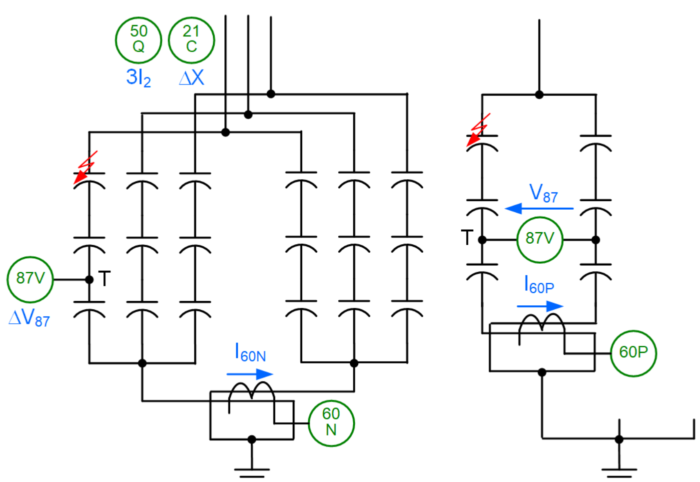
All values are in per unit.

Voltage and current phase angles are relative to the faulted-phase voltage.

For internal faults, the 59NU operating signal is the same as the 59N operating signal.

Grounded Double-Wye Capacitor Bank

Fail-Open	Fail-Short
$k_{OV} = \frac{SP}{SP - F(S-1)}$ $F = \frac{SP}{S-1}\frac{k_{OV}-1}{k_{OV}}$	$k_{OV} = \frac{S}{S-F}$ $F = S\frac{k_{OV}-1}{k_{OV}}$
$3I_2 = \frac{1}{2}\frac{F}{SPR - F(SR-R)}1\angle{-90°}$ $3I_2 = \frac{k_{OV}-1}{2R(S-1)}1\angle{-90°}$	$3I_2 = \frac{1}{2}\frac{F}{SR - FR}1\angle 90°$ $3I_2 = \frac{k_{OV}-1}{2R}1\angle 90°$
$\Delta X_{PHASE} = \frac{F}{2SRP - F(2SR - 2R + 1)}$ $\Delta X_{PHASE} = \frac{k_{OV}-1}{2R(S-1) + 1 - k_{OV}}$	$\Delta X_{PHASE} = -\frac{F}{2SR - F(2R-1)}$ $\Delta X_{PHASE} = -\frac{k_{OV}-1}{2R + k_{OV} - 1}$
$\Delta X_{STRING} = 2R\frac{F}{SP - FS}$ $\Delta X_{STRING} = 2R\frac{k_{OV}-1}{S - k_{OV}}$	$\Delta X_{STRING} = -\frac{2R}{S}F$ $\Delta X_{STRING} = -2R\frac{k_{OV}-1}{k_{OV}}$
$\Delta V_{87} = V_{87} = \frac{TF}{SRP - F\left(SR - \frac{R-T}{1-T}\right)}1\angle 0°$ $\Delta V_{87} = V_{87} = \frac{T(1-T)(k_{OV}-1)}{R(S-1) - T(SR - (R-1)k_{OV} - 1)}1\angle 0°$	$\Delta V_{87} = V_{87} = \frac{TF}{SR - F\frac{R-T}{1-T}}1\angle 180°$ $\Delta V_{87} = V_{87} = \frac{T(1-T)(k_{OV}-1)}{R - T\left((R-1)k_{OV} + 1\right)}1\angle 180°$
$I_{60P} = I_{60N} = 3I_2$	



Notes:

All values are in per unit.

ΔX_{PHASE} and ΔX_{STRING} are both in per unit of the bank reactance.

Voltage and current phase angles are relative to the faulted-phase voltage.

Δ87V differential signal uses bus voltage scaled down to the tap voltage ($V_{TAP} - T \cdot V_{BUS}$).

$$T = \frac{X_{BOTTOM}}{X_{TOP} + X_{BOTTOM}}$$

For nonhomogeneous banks (different unit arrangement above and below the tap), P and R are parameters of the top part and S is an equivalent value, as follows:

$$S = \frac{S_{TOP}}{1-T}$$

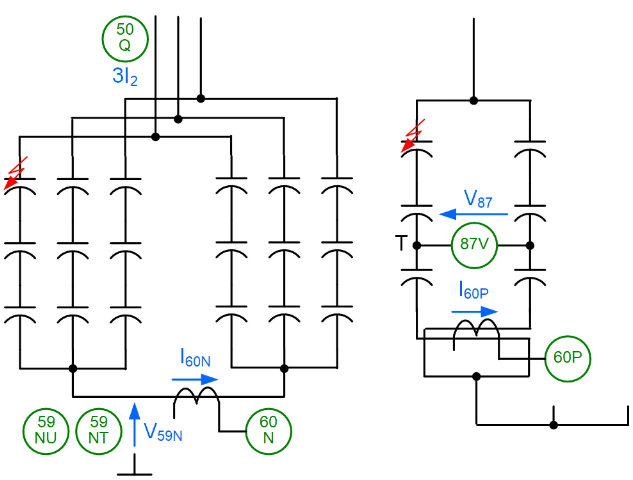
UNGROUNDED DOUBLE-WYE CAPACITOR BANK

Fail-Open	Fail-Short
$k_{OV} = \dfrac{6SPR}{6SPR - F(6SR - 6R + 1)}$ $F = \dfrac{6SPR}{6SR - 6R + 1}\dfrac{k_{OV} - 1}{k_{OV}}$	$k_{OV} = \dfrac{6SR}{6SR - F(6R - 1)}$ $F = \dfrac{6SR}{6R - 1}\dfrac{k_{OV} - 1}{k_{OV}}$
$3I_2 = \dfrac{3F}{6SPR - F(6SR - 6R + 1)} 1\angle{-90°}$ $3I_2 = 3\dfrac{k_{OV} - 1}{6R(S - 1) + 1} 1\angle{-90°}$	$3I_2 = \dfrac{3F}{6SR - F(6R - 1)} 1\angle 90°$ $3I_2 = 3\dfrac{k_{OV} - 1}{6R - 1} 1\angle 90°$
$V_{59N} = \dfrac{F}{6SPR - F(6SR - 6R + 1)} 1\angle 180°$ $V_{59N} = \dfrac{k_{OV} - 1}{6R(S - 1) + 1} 1\angle 180°$	$V_{59N} = \dfrac{F}{6SR - F(6R - 1)} 1\angle 0°$ $V_{59N} = \dfrac{k_{OV} - 1}{6R - 1} 1\angle 0°$
$V_{87} = \dfrac{6TF}{6SRP - F\left(6SR - \dfrac{6R - 5T - 1}{1 - T}\right)} 1\angle 0°$ $V_{87} = \dfrac{6T(1 - T)(k_{OV} - 1)}{6R(S - 1) - T(6SR - 6(R - 1)k_{OV} - 5) + 1} 1\angle 0°$	$V_{87} = \dfrac{6TF}{6SR - F\dfrac{6R - 5T - 1}{1 - T}} 1\angle 180°$ $V_{87} = \dfrac{6T(1 - T)(k_{OV} - 1)}{6R - T(6(R - 1)k_{OV} + 5) - 1} 1\angle 180°$

$$I_{60P} = 2I_{60N} = 3I_2$$

$$I_{60N} = \frac{1}{2} 3I_2$$

$$V_{59N} = -j\frac{1}{3} 3I_2$$



Notes:

All values are in per unit.

Voltage and current phase angles are relative to the faulted-phase voltage.

$$T = \frac{X_{BOTTOM}}{X_{TOP} + X_{BOTTOM}}$$

For nonhomogeneous banks (different unit arrangement above and below the tap), P and R are parameters of the top part and S is an equivalent value, as follows:

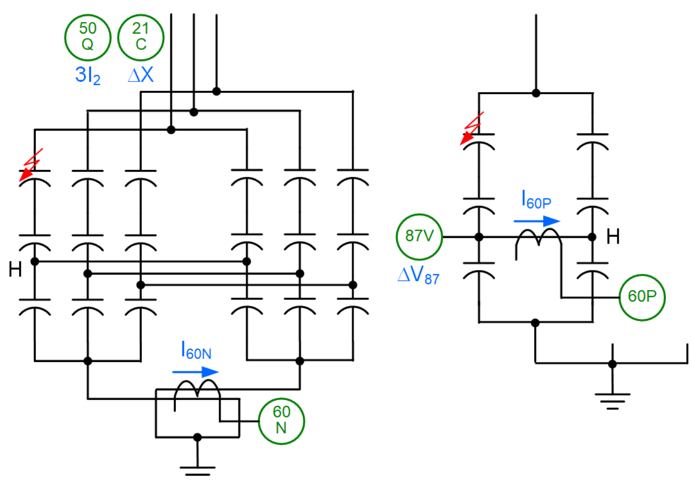
$$S = \frac{S_{TOP}}{1 - T}$$

GROUNDED H-BRIDGE CAPACITOR BANK

Fail-Open	Fail-Short
$k_{OV} = \frac{2SPRG}{2SPRG - F(2SRG - 2R + 1 - G)}$ $F = \frac{2SPRG}{2SRG - 2R + 1 - G}\frac{k_{OV} - 1}{k_{OV}}$	$k_{OV} = \frac{2SRG}{2SRG - F(2R - 1 + G)}$ $F = \frac{2SRG}{2R - 1 + G}\frac{k_{OV} - 1}{k_{OV}}$
$3I_2 = \frac{GF}{2SPRG - F(2SRG - 2R + 1 - G)} 1\angle{-90°}$ $3I_2 = \frac{G(k_{OV} - 1)}{2SRG - 2R + 1 - G} 1\angle{-90°}$	$3I_2 = \frac{GF}{2SRG - F(2R - 1 + G)} 1\angle 90°$ $3I_2 = \frac{G\,(k_{OV} - 1)}{2R - 1 + G} 1\angle 90°$
$\Delta X_{PHASE} = \frac{GF}{2SRPG - F(2SRG - 2R + 1)}$ $\Delta X_{PHASE} = \frac{G(k_{OV} - 1)}{2SRG - 2R + 1 - Gk_{OV}}$	$\Delta X_{PHASE} = -\frac{GF}{2SRG - F(2R - 1)}$ $\Delta X_{PHASE} = -\frac{G\,(k_{OV} - 1)}{2R + Gk_{OV} - 1}$
$\Delta V_{87} = \frac{HGF}{4SRPG - F(4SRG - 4R + 2)} 1\angle 180°$ $\Delta V_{87} = \frac{HG(k_{OV} - 1)}{2(2SRG - 2R + 1 - Gk_{OV})} 1\angle 180°$	$\Delta V_{87} = \frac{HGF}{4SRG - F(4R - 2)} 1\angle 0°$ $\Delta V_{87} = \frac{HG\,(k_{OV} - 1)}{2(2R + Gk_{OV} - 1)} 1\angle 0°$
$I_{60P} = \frac{1}{2}\frac{F}{2SPRG - F(2SRG - 2R + 1 - G)} 1\angle{-90°}$ $I_{60P} = \frac{1}{2}\frac{k_{OV} - 1}{2SRG - 2R + 1 - G} 1\angle{-90°}$	$I_{60P} = \frac{1}{2}\frac{F}{2SRG - F(2R - 1 + G)} 1\angle 90°$ $I_{60P} = \frac{1}{2}\frac{k_{OV} - 1}{2R - 1 + G} 1\angle 90°$

$$I_{60P} = \frac{1}{2G} 3I_2$$

$$I_{60N} = \begin{cases} 0 & \text{for failures above the bridge} \\ 2I_{60P} & \text{for failures below the bridge} \end{cases}$$



Notes:

All values are in per unit.

Voltage and current phase angles are relative to the faulted-phase voltage.

H is the per-unit position of the bridge relative to the neutral point.

For failures above the bridge, use G = 1 – H. For failures below the bridge, use G = H and add 180° to the phase angle. The 180° angle shift does not apply to $3I_2$ and ΔX.

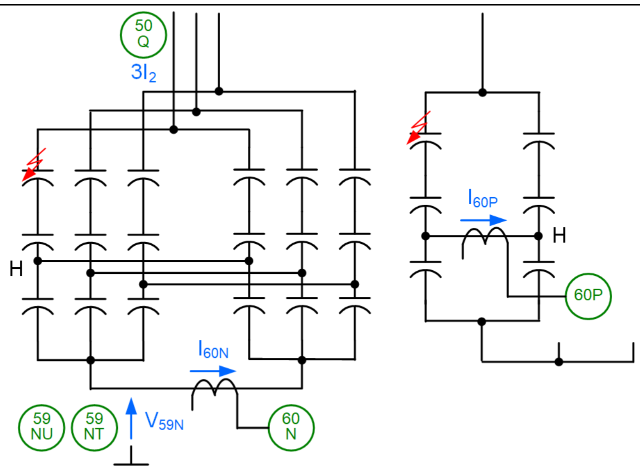
UNGROUNDED H-BRIDGE CAPACITOR BANK

Fail-Open	Fail-Short
$k_{OV} = \frac{6SPRG}{6SPRG - F(6SRG - 6R + 3 - 2G)}$ $F = \frac{6SPRG}{6SRG - 6R + 3 - 2G} \frac{k_{OV} - 1}{k_{OV}}$	$k_{OV} = \frac{6SRG}{6SRG - F(6R - 3 + 2G)}$ $F = \frac{6SRG}{6R - 3 + 2G} \frac{k_{OV} - 1}{k_{OV}}$
$3I_2 = \frac{3GF}{6SPRG - F(6SRG - 6R + 3 - 2G)} 1\angle{-90°}$ $3I_2 = \frac{3G(k_{OV} - 1)}{6SRG - 6R + 3 - 2G} 1\angle{-90°}$	$3I_2 = \frac{3GF}{6SRG - F(6R - 3 + 2G)} 1\angle 90°$ $3I_2 = \frac{3G(k_{OV} - 1)}{6R - 3 + 2G} 1\angle 90°$
$V_{59N} = \frac{GF}{6SPRG - F(6SRG - 6R + 3 - 2G)} 1\angle 180°$ $V_{59N} = \frac{G(k_{OV} - 1)}{6SRG - 6R + 3 - 2G} 1\angle 180°$	$V_{59N} = \frac{GF}{6SRG - F(6R - 3 + 2G)} 1\angle 0°$ $V_{59N} = \frac{G(k_{OV} - 1)}{6R - 3 + 2G} 1\angle 0°$
$I_{60P} = \frac{3F}{6SPRG - F(6SRG - 6R + 3 - 2G)} 1\angle{-90°}$ $I_{60P} = \frac{3(k_{OV} - 1)}{6SRG - 6R + 3 - 2G} 1\angle{-90°}$	$I_{60P} = \frac{3F}{6SRG - F(6R - 3 + 2G)} 1\angle 90°$ $I_{60P} = \frac{3(k_{OV} - 1)}{6R - 3 + 2G} 1\angle 90°$

$$I_{60P} = \frac{1}{G} 3I_2$$

$$I_{60N} = I_{60P}$$

$$V_{59N} = -j\frac{1}{3} 3I_2$$



Notes:

All values are in per unit.

Voltage and current phase angles are relative to the faulted-phase voltage.

H is the per-unit position of the bridge relative to the neutral point.

For failures above the bridge, use G = 1 – H. For failures below the bridge, use G = H and add 180° to the phase angle. The 180° angle shift does not apply to $3I_2$ and ΔX.

X. Appendix B. Unbalance Calculations for Multiple Failures

In Section VI, we explained how to perform unbalance calculations for a failure that may involve many capacitor units but that occurred at a single location. In this appendix, we explain how to apply the superposition principle to perform calculations for failures at multiple locations. Using this approach, you can perform calculations for evolving and equalizing failures and for failures under a pre-existing unbalance.

Capacitor unit failures create only small changes in the bank voltages and currents. Equations that tie the capacitor bank voltages and currents are linear. Therefore, we can use the superposition principle to solve the bank equations for multiple failures.

We can think of a derivation of an unbalance equation as an application of the Thevenin principle. An unbalance protection operating signal is a superimposed quantity in the Thevenin method. The system sources are removed (shorted) as per the Thevenin principle and the change in voltage at the failure location drives the unbalance signal.

Because a unit failure does not greatly affect the voltages inside the capacitor bank, we can assume nominal conditions and neglect the previous failure(s) when calculating the unbalance protection signals for the subsequent failure. Therefore, simultaneous failures at multiple locations can be treated as a superposition of a set of single failures, where each single failure occurs separately and under nominal conditions.

Use the following procedure to perform unbalance calculations for multiple failures:

1. Calculate the unbalance protection signal of interest separately for each failure, each time assuming a healthy bank prior to the failure (use the equations in Appendix A).
2. Observing the measuring polarity convention, failure location (phase, above or below the bridge, etc.), and the unbalance signal phase angle, add (vectorially, as phasors) the signals obtained separately for each failure and obtain the unbalance signal for multiple failures.

The above approach is not perfectly accurate but is accurate enough for practical engineering calculations, especially when considering failures that do not result in very large voltage or current changes in the bank.

<u>Example B.1</u>

Let us continue Example 2 from Subsection V.B and calculate the V_{59N} operating signal for the following failure scenario: 1) two units fail open in the A-phase, 2) the protection system issues an alarm but does not trip, and 3) a single unit fails in the B-phase before the bank can be repaired.

The V_{59N} signal for the first failure (A-phase, F = 2) is:

$$V_{59N} = \frac{2 \cdot 1\angle 180°}{3 \cdot 4 \cdot 14 \cdot 1 - 2 \cdot (3 \cdot 4 \cdot 1 - 3 \cdot 1 + 1)} = \cdots$$

$$\ldots = 0.01351 \text{ pu } \angle 180°$$

The V_{59N} signal for the second failure (B-phase, F = 1) is:

$$V_{59N} = \frac{1 \cdot 1\angle(180° - 120°)}{3 \cdot 4 \cdot 14 \cdot 1 - 1 \cdot (3 \cdot 4 \cdot 1 - 3 \cdot 1 + 1)} = \cdots$$

$$\ldots = 0.006329 \text{ pu } \angle 60°$$

The V_{59N} phase angle is relative to the faulted-phase voltage. Therefore, we subtracted 120° when calculating the V_{59N} operating signal for the failure in the B-phase.

The V_{59N} operating signal with both failures present is:

$$V_{59N} = 0.01351 \text{ pu } \angle 180° + 0.006329\text{pu } \angle 60° = \cdots$$

$$\ldots = 0.01171 \text{ pu } \angle 152°$$

The V_{59N} operating signal decreased from 0.01351 pu after the first failure to 0.01171 pu after the second failure. This is expected because the second failure equalized the bank to some extent.

Assume next that the second failure involves another unit (F = 2 in the B-phase). We can recalculate the B-phase failure by using F = 2, or we can consider the second unit failure in the B-phase as the third failure in the bank. If we follow the latter approach, we can calculate the V_{59N} operating signal as follows:

$$V_{59N} = 0.01171 \text{ pu } \angle 152° + 0.006329 \text{ pu } \angle 60° = \cdots$$

$$\ldots = 0.01311 \text{ pu } \angle 123°$$

<u>Example B.2</u>

Consider the grounded double-wye externally fused capacitor bank in Subsection VI.C (P = 15, S = 6, and R = 1), and calculate the negative-sequence ($3I_2$) and the neutral unbalance (I_{60N}) currents for the following evolving failure: 1) one unit fails in the A-phase of the left half of the bank, 2) followed by another unit failure in the A-phase of the right half of the bank, 3) followed by another unit failure in the B-phase of the left half of the bank (Fig. B.1).

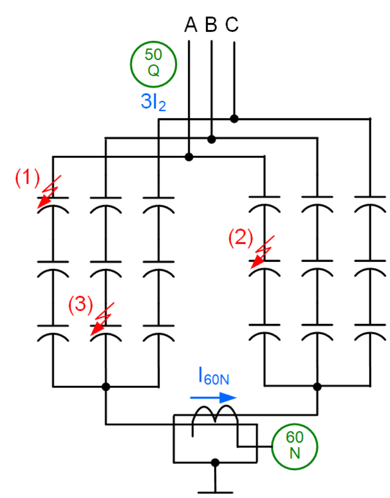


Fig. B.1. Progressing failure in the grounded double-wye bank in Example B.2.

We use equations from Appendix A for the fail-open scenario (fused bank) and separately calculate the operating signal components resulting from each failure.

First unit failure in the A-phase of the left half of the bank:

$$3I_{2(1)} = \frac{1}{2} \cdot \frac{1}{6 \cdot 15 \cdot 1 - 1 \cdot (6 \cdot 1 - 1)} 1\angle{-90°} = \cdots$$

$$\ldots = 0.00588241 \text{ pu } \angle{-90°}$$

$$I_{60N(1)} = 3I_{2(1)} = 0.00588241 \text{ pu } \angle{-90°}$$

Second unit failure in the A-phase of the right half of the bank (we invert the I_{60N} polarity to account for the I_{60N} measuring convention from the left to the right half of the bank):

$$3I_{2(2)} = 3I_{2(1)} = 0.00588241 \text{ pu } \angle{-90°}$$

$$I_{60N(2)} = -I_{60N(1)} = 0.00588241 \text{ pu } \angle 90°$$

Third unit failure in the B-phase of the left half of the bank (we subtract 120° to account for the B-phase being faulted):

$$3I_{2(3)} = 3I_{2(1)} \cdot 1\angle{-120°} = 0.00588241 \text{ pu } \angle 150°$$

$$I_{60N(3)} = I_{60N(1)} \cdot 1\angle{-120°} = 0.00588241 \text{ pu } \angle 150°$$

We apply the principle of superposition and sum the above operating signal components (vectorially, as phasors) and obtain the following magnitudes of the two operating signals for the three stages of the failure:

Per-Unit Unbalance Protection Signal After Failures

	First Failure	Second Failure	Third Failure
$3I_2$	0.00588241	0.01176482	0.01556339
I_{60N}	0.00588241	0.00000000	0.00588241

The $3I_2$ operating signal increases after each failure because, from the point of view of the bank terminals, the bank becomes more unbalanced after each failure. The I_{60N} operating signal goes back to zero after the second failure. This is because at that time, the left and right halves of the bank are symmetrical – they both have a single unit failure in the A-phase. When, subsequently, the left half of the bank suffers another failure in the B-phase, the I_{60N} signal increases again to a value as for a single unit failure. This is because at that time, the difference between the failed units in the left and right halves of the bank is 2 – 1 = 1.

This example shows that even though the $3I_2$ and I_{60N} signals are identical for a single unit failure, when multiple failures occur at different locations in the bank, the $3I_2$ and I_{60N} signals may differ depending on how the failures equalize inside the bank.

Example B.3

Consider the grounded H-bridge fuseless capacitor bank in Subsection VI.D (P = 1, S = 22, R = 2, and H = 0.5 pu), and calculate the phase differential voltage (ΔV_{87}) and the phase unbalance current (I_{60P}) for the following evolving failure: 1) one unit fails in the A-phase above the bridge in the left half of the bank, 2) followed by another unit failure in the A-phase below the bridge in the left half of the bank, 3) followed by another unit failure in the A-phase above the bridge in the right half of the bank (Fig. B.2).

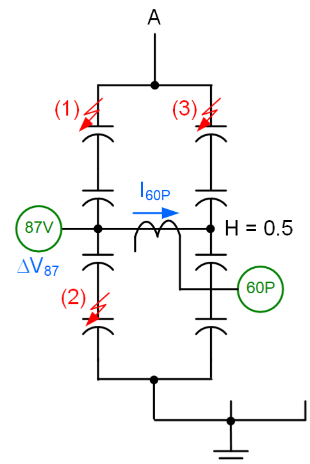


Fig. B.2. Progressing failure in the grounded H-bridge bank in Example B.3.

We use equations from Appendix A for the fail-short scenario (fuseless bank) and separately calculate the operating signal components resulting from each failure.

First unit failure in the A-phase above the bridge in the left half of the bank (G = 1 – H = 0.5):

$$\Delta V_{87(1)} = \frac{0.5 \cdot 0.5 \cdot 1}{4 \cdot 22 \cdot 2 \cdot 0.5 - 1 \cdot (4 \cdot 2 - 2)} 1\angle 0° = \cdots$$

$$\ldots = 0.00304878 \text{ pu } \angle 0°$$

$$I_{60P(1)} = \frac{1}{2} \cdot \frac{1}{2 \cdot 22 \cdot 2 \cdot 0.5 - 1 \cdot (2 \cdot 2 - 1 + 0.5)} 1\angle 90° = \cdots$$

$$\ldots = 0.01234567 \text{ pu } \angle 90°$$

Second unit failure in the A-phase below the bridge in the left half of the bank (we invert the polarity to account for the failure below the bridge, G = H = 0.5):

$$\Delta V_{87(2)} = \frac{0.5 \cdot 0.5 \cdot 1}{4 \cdot 22 \cdot 2 \cdot 0.5 - 1 \cdot (4 \cdot 2 - 2)} 1\angle 180° = \cdots$$

$$\ldots = 0.00304878 \text{ pu } \angle 180°$$

$$I_{60P(2)} = \frac{1}{2} \cdot \frac{1}{2 \cdot 22 \cdot 2 \cdot 0.5 - 1 \cdot (2 \cdot 2 - 1 + 0.5)} 1\angle{-90°} = \cdots$$

$$\ldots = 0.01234567 \text{ pu } \angle{-90°}$$

Third unit failure in the A-phase above the bridge in the right half of the bank (we invert the I_{60P} polarity to account for the I_{60P} measuring convention from the left to the right half of the bank, G = 1 – H = 0.5):

$$\Delta V_{87(3)} = \frac{0.5 \cdot 0.5 \cdot 1}{4 \cdot 22 \cdot 2 \cdot 0.5 - 1 \cdot (4 \cdot 2 - 2)} 1\angle 0° = \cdots$$

$$\ldots = 0.00304878 \text{ pu } \angle 0°$$

$$I_{60P(3)} = \frac{1}{2} \cdot \frac{1}{2 \cdot 22 \cdot 2 \cdot 0.5 - 1 \cdot (2 \cdot 2 - 1 + 0.5)} 1\angle{-90°} = \cdots$$

$$\ldots = 0.01234567 \text{ pu } \angle{-90°}$$

We apply the principle of superposition and sum the above operating signal components (vectorially, as phasors) and obtain the following magnitudes of the two operating signals for the three stages of the failure:

Per-Unit Unbalance Protection Signal After Failures

	First Failure	Second Failure	Third Failure
ΔV_{87}	0.00304878	0.000000	0.00304878
I_{60P}	0.01234567	0.000000	0.01234567

The second failure (one unit failed above and one unit failed below the bridge in the A-phase of the left bank) makes the parts above and below the bridge of the A-phase of the bank symmetrical. Therefore, the operating signals of both the 87V and 60P elements drop to zero. The third failure makes the A-phase unbalanced (two units failed above the bridge and one unit failed below the bridge), and the two operating signals increase again.

The $3I_2$ operating signal increases after each failure because, from the point of view of the bank terminals, the bank becomes more unbalanced after each failure. The I_{60N} operating signal also increases after each failure because, from the point of view of the neutral current, the A-phase of the bank becomes more unbalanced after each failure compared with the B- and C-phases.

Example B.2 and Example B.3 show how multiple protection elements can become blind to equalizing bank failures. It is beneficial to use several unbalance protection elements to mutually cover their weak spots during equalizing failures.

XI. Appendix C. Unbalance Calculations for Partial Capacitor Unit Failures

In the main body of the paper, we assumed that a failure affects the entire capacitor unit or multiple units. In this appendix, we introduce a simple method for unbalance calculations at the capacitor element level (partial unit failure). We represent a partially failed unit by using a fractional value of F. This approach allows us to use the unbalance equations derived for failed units to perform unbalance calculations for failures of capacitor elements.

When an element fails short in a fuseless or externally fused bank, the unit reactance reduces slightly. Similarly, when a fuse blows in an internally fused bank, the unit reactance increases slightly. We calculate an equivalent fractional failure size, F_{FRAC}, that represents the change in the unit reactance because of the element failure(s) and use this fractional failure size, F_{FRAC}, in the unbalance calculations.

This method is very convenient because it decouples the arrangement of the bank (units, groups, strings, phases, and phase connections) and the arrangement of the capacitor unit (capacitor elements and fuses). Moreover, our method allows unbalance calculations for banks that include capacitor units of different types.

A. Partial Fail-Open Scenario (Internally Fused Banks)

In the unit fail-open scenario (Fig. 9), the group reactance changes as follows:

$$X_{GROUP}: \ \frac{X_U}{P} \rightarrow \frac{X_U}{P-F} \tag{C.1}$$

Assume a partial unit failure (some but not all internal fuses in the unit are blown). This partial failure increases the unit reactance from the nominal value of X_U to some other value, X_{UF}. The affected group now comprises (P – 1) healthy units and one unit that has a different reactance. The equivalent reactance of the group is:

$$\frac{X_U \cdot X_{UF}}{X_U + (P-1) \cdot X_{UF}} \tag{C.2}$$

We equate the group reactance from our failure model (C.1) and the actual group reactance (C.2):

$$\frac{X_U}{P - F_{FRAC}} = \frac{X_U \cdot X_{UF}}{X_U + (P-1) \cdot X_{UF}} \tag{C.3}$$

We solve (C.3) to obtain the equivalent partial failure size, F_{FRAC}, as follows:

$$F_{FRAC} = 1 - \frac{X_U}{X_{UF}} = 1 - \alpha, \qquad \alpha = \frac{X_U}{X_{UF}} \tag{C.4}$$

You can perform unbalance calculations for fail-open capacitor element failures by using the fractional failure size, F_{FRAC}, obtained by using (C.4).

If the capacitor unit fails open completely, then $X_{UF} = \infty$ and $F_{FRAC} = 1$, as expected. If there is no failure, then $X_{UF} = X_U$ and $F_{FRAC} = 0$, as expected.

B. Partial Fail-Short Scenario (Fuseless and Externally Fused Banks)

In the unit fail-short scenario (Fig. 10), the string reactance changes as follows:

$$X_{STRING}: \ \frac{S}{P} \cdot X_U \rightarrow \frac{S-F}{P} \cdot X_U \tag{C.5}$$

Assume a partial unit failure (a short-circuit of some but not all capacitor element groups inside the capacitor unit). This partial failure decreases the unit reactance from the nominal value of X_U to some other value, X_{UF}. The affected string now comprises (S – 1) healthy groups and one group that has a different reactance. That group has P – 1 healthy capacitor units and one partially failed unit. Therefore, the equivalent reactance of the string is:

$$\frac{S-1}{P} \cdot X_U + \frac{X_U \cdot X_{UF}}{X_U + (P-1) \cdot X_{UF}} \tag{C.6}$$

We equate the string reactance from our failure model (C.5) and the actual string reactance (C.6):

$$\frac{S - F_{FRAC}}{P} \cdot X_U = \frac{S-1}{P} \cdot X_U + \frac{X_U \cdot X_{UF}}{X_U + (P-1) \cdot X_{UF}} \tag{C.7}$$

We solve (C.7) to obtain the equivalent partial failure size, F_{FRAC}, as follows:

$$F_{FRAC} = \frac{X_U - X_{UF}}{X_U + (P-1) \cdot X_{UF}} = \frac{\alpha - 1}{\alpha - 1 + P} \qquad (C.8)$$

You can perform unbalance calculations for fail-short element failures by using the fractional failure size, F_{FRAC}, obtained by using (C.8).

If the capacitor unit fails short completely, then $X_{UF} = 0$ and $F_{FRAC} = 1$, as expected. If there is no failure, then $X_{UF} = X_U$ and $F_{FRAC} = 0$, as expected.

C. Unbalance Calculations for Partial Unit Failures

Apply the following procedure to perform unbalance calculations for capacitor element failures (partial capacitor unit failures):

1. Calculate the nominal (X_U) capacitor unit reactance and the reactance of the partially failed unit (X_{UF}). Because only the ratio (α) of the two reactances matters, these calculations are very simple and can be done by hand by just inspecting the internal connection diagram of the capacitor unit and counting the capacitor elements (see Example C.1 and Example C.2).
2. Calculate the fractional failure size, F_{FRAC}, by using (C.4) for internally fused banks and (C.8) for externally fused and fuseless banks.
3. Use the unbalance equations in Appendix A to calculate the unbalance signals of interest based on the fractional failure size, F_{FRAC}.
4. Use the superposition principle described in Appendix B to perform calculations for partial and complete unit failures, including cases where the two failures occur at different locations in the bank.

Example C.1

Consider the ungrounded single-wye capacitor bank in the setting calculation example in Subsection VI.B. The bank is externally fused. Assume that each capacitor unit has 5 capacitor elements in parallel in a group and 8 groups connected in series (Fig. C.1). Calculate the 59N element operating signal for the case where 2 groups of capacitor elements fail short inside an A-phase capacitor unit.

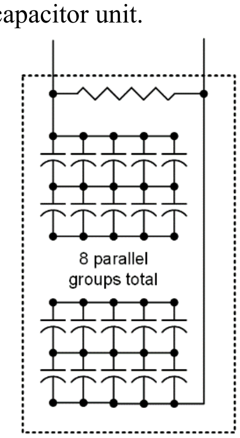


Fig. C.1. Externally fused capacitor unit in Example C.1.

The healthy capacitor unit reactance is 8/5 of the capacitor element reactance. When two capacitor element groups in a capacitor unit are shorted, the capacitor unit reactance becomes 6/5 of the capacitor element reactance. Therefore, $\alpha = (8/5) / (6/5) = 8/6$. We apply (C.8) and calculate the fractional failure size, F_{FRAC}:

$$F_{FRAC} = \frac{8/6 - 1}{8/6 - 1 + 14} = 0.0233$$

Note that the shorting of two capacitor element groups removes 10 capacitor elements out of the total of 50. The calculated fractional failure size of 0.0233 is close to the simple proportion of 10/50 = 0.025. This observation leads to an opportunity of using the ratio of removed capacitor elements and the total number of elements as an approximation of the fractional unit failure.

Next, we use the equation for the 59N operating signal in an ungrounded single-wye bank for the fail-short scenario and calculate:

$$V_{59N} = \frac{0.0233}{3 \cdot 4 \cdot 1 - 0.0233 \cdot (3 \cdot 14 - 1)} = \cdots$$

$$\ldots = 0.0021 \text{ pu} = 0.504 \text{ V sec}$$

For comparison, when we assumed that an entire unit failed open (F = 1), we obtained $V_{59N} = 0.0063291$ pu. When the entire unit fails open, the V_{59N} operating signal is out of phase with the faulted-phase voltage; when there is a partial fail-short failure in the unit, the V_{59N} operating signal is in phase with the faulted-phase voltage (see Appendix A). When the capacitor elements inside the capacitor unit fail short in a cascading fashion, there is a phase inversion of the unbalance protection operating signals at a time when the progressing fail-short capacitor element failure becomes a permanent fail-open capacitor unit failure because of the operation of the external fuse.

Assume you want to calculate the 59N operating signal for the case when one unit failed open (F = 1) and some other unit failed short partially (F_{FRAC}). You can use the principle of superposition and calculate the V_{59N} signal separately for the two failures (F = 1, fail open, and F = F_{FRAC}, fail short). To obtain the final V_{59N} value, you must add (vectorially, as phasors) the two V_{59N} signal components.

Example C.2

Consider the grounded double-wye capacitor bank in the setting calculation example in Subsection VI.C, but assume the bank is internally fused (unlike in Subsection VI.C). Assume each capacitor unit has 15 capacitor elements in parallel in a group and 5 groups connected in series (Fig. C.2). Calculate the 60N element operating signal for the case where 4 capacitor elements fail open in a single group inside a capacitor unit in the A-phase of the left half of the bank.

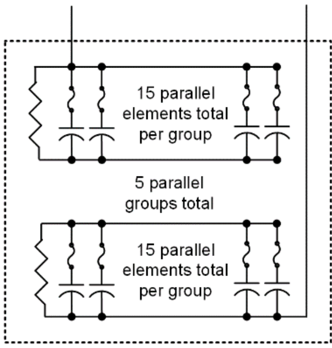


Fig. C.2. Internally fused capacitor unit in Example C.2.

The healthy capacitor unit reactance is 5/15 of the capacitor element reactance. When 4 capacitor elements fail open in a group inside the capacitor unit, the capacitor unit reactance becomes 1/(15 – 4) + 4/15 = 59/165 of the element reactance. Therefore, α = (5/15) / (59/165) = 0.9322. We apply (C.4) and calculate the fractional failure size, F_{FRAC}:

$$F_{FRAC} = 1 - 0.9322 = 0.0678$$

Note that the failure removes 4 capacitor elements out of the total of 75. The calculated fractional failure size of 0.0678 is close to the simple proportion of 4/75 = 0.0533. The two values are not identical because the capacitor element failures redistribute the voltage inside the unit and affect the unit apparent reactance.

Next, we use the equation for the 60N operating signal in a grounded double-wye bank for the fail-open scenario and calculate:

$$I_{60N} = \frac{1}{2} \cdot \frac{0.0678}{6 \cdot 15 \cdot 1 - 0.0678 \cdot (6 \cdot 1 - 1)} = \cdots$$

$$\ldots = 0.000378 \text{ pu}$$

For comparison, when we assumed that an entire unit failed open (F = 1), we obtained I_{60N} = 0.005882 pu.

Let us go back to (54) and approximate the I_{60N} signal based on the number of capacitor elements lost because of the failure. The bank has 15 · 5 = 75 capacitor elements in a capacitor unit and 15 · 6 · 1 · 2 = 180 units in a phase, for a total of 75 · 180 = 13,500 capacitor elements in a phase. When 4 capacitor elements fail open (are removed), the per-unit unbalance can be approximated as 4/13,500 = 0.0002962 pu (a 20 percent error compared with the accurate value of 0.000378 pu). When the entire capacitor unit fails open, the per-unit unbalance can be approximated as 1/180 = 0.005555 pu (a 5 percent error compared with the accurate value of 0.005882 pu).

XII. References

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XIII. Biographies

Bogdan Kasztenny has over 30 years of experience in power system protection and control. In his decade-long academic career (1989–1999), Dr. Kasztenny taught power system and digital signal processing courses at several universities and conducted applied research for several relay manufacturers. In 1999, Bogdan left academia for relay manufacturers where he has since designed, applied, and supported protection, control, and fault-locating products with their global installations numbering in the thousands. Bogdan is an IEEE Fellow, a Senior Fulbright Fellow, a Distinguished CIGRE Member, and a registered professional engineer in the province of Ontario. Bogdan has served as a Canadian representative of the CIGRE Study Committee B5 (2013–2020) and on the Western Protective Relay Conference Program Committee (2011–2020). In 2019, Bogdan received the IEEE Canada P. D. Ziogas Electric Power Award. Bogdan earned both the Ph.D. (1992) and D.Sc. (Dr. habil., 2019) degrees, has authored over 220 technical papers, holds over 55 U.S. patents, and is an associate editor of the *IEEE Transactions on Power Delivery*.

Satish Samineni received his bachelor of engineering degree in electrical and electronics engineering from Andhra University, Visakhapatnam, India, in 2000. He received his master's degree in electrical engineering in 2003 and a Ph.D. in 2021 from the University of Idaho. Since 2003, he has been with Schweitzer Engineering Laboratories, Inc. in Pullman, Washington, where he is a principal research engineer in the research and development division. He has authored or coauthored several technical papers and holds multiple U.S. patents. His research interests include power system protection, power system modeling, power electronics and drives, synchrophasor-based control applications, and power system stability. He is a registered professional engineer in the state of Washington and a senior member of IEEE.