Tutorial on Symmetrical Components

Part 1: Examples

Ariana Amberg and Alex Rangel, *Schweitzer Engineering Laboratories, Inc.*

***Abstract*—Symmetrical components and the per-unit system are two of the most fundamental and necessary types of mathematics for relay engineers and technicians. We must practice these techniques in order to fully understand and feel comfortable with them. This white paper provides both theoretical and real-world examples with questions that can be used to gain experience with symmetrical components. The full solutions to these questions can be found in the white paper "Tutorial on Symmetrical Components, Part 2: Solutions," available for download at http://www.selinc.com.**

I. Introduction

The method of symmetrical components is used to simplify fault analysis by converting a three-phase unbalanced system into two sets of balanced phasors and a set of single-phase phasors, or symmetrical components. These sets of phasors are called the positive-, negative-, and zero-sequence components. These components allow for the simple analysis of power systems under faulted or other unbalanced conditions. Once the system is solved in the symmetrical component domain, the results can be transformed back to the phase domain.

The topic of symmetrical components is very broad and can take considerable time to cover in depth. A summary of important points is included in this introduction, although it is highly recommended that other references be studied for a more thorough explanation of the mathematics involved. Refer to [1], [2], [3], [4], and [5] for more information on symmetrical components.

A. Converting Between the Phase and Symmetrical Component Domains

Any set of phase quantities can be converted into symmetrical components, where α is defined as $1\angle 120$, as follows:

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} \tag{1}$$

where I_0, I_1, and I_2 are the zero-, positive-, and negative-sequence components, respectively. This equation shows the symmetrical component transformation in terms of currents, but the same equations are valid for voltages as well.

This results in the following equations:

$$\begin{aligned} I_0 &= \frac{1}{3}\left(I_A + I_B + I_C\right) \\ I_1 &= \frac{1}{3}\left(I_A + \alpha I_B + \alpha^2 I_C\right) \\ I_2 &= \frac{1}{3}\left(I_A + \alpha^2 I_B + \alpha I_C\right) \end{aligned} \tag{2}$$

Likewise, a set of symmetrical components can be converted into phase quantities as follows:

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} \tag{3}$$

This results in the following equations:

$$\begin{aligned} I_A &= I_0 + I_1 + I_2 \\ I_B &= I_0 + \alpha^2 I_1 + \alpha I_2 \\ I_C &= I_0 + \alpha I_1 + \alpha^2 I_2 \end{aligned} \tag{4}$$

These conversions are valid for an A-phase base, which can be used for A-phase-to-ground, B-phase-to-C-phase, B-phase-to-C-phase-to-ground, and three-phase faults. In Section V, Example 4 shows how the base changes for other irregular fault types. These conversions are also only valid for an ABC system phase rotation. In Section VI, Example 5 shows how the equations change for an ACB system phase rotation.

A calculator was created in Microsoft® Excel® to allow us to convert between the phase and symmetrical component domains. This calculator is available for download with this white paper at http://www.selinc.com.

B. Transformer Representations in the Sequence Networks

For information on the formation of the sequence networks as well as the representation of power system components in the sequence networks, see [1] and [2].

Transformers are simply represented as their positive- and negative-sequence impedances in the positive- and negative-sequence networks, respectively. However, the transformer representation in the zero-sequence network can be more complex and is dependent on the type of transformer connection. Fig. 1 shows some common transformer connections and the equivalent zero-sequence representations. For a complete list of transformer connections, see [1].

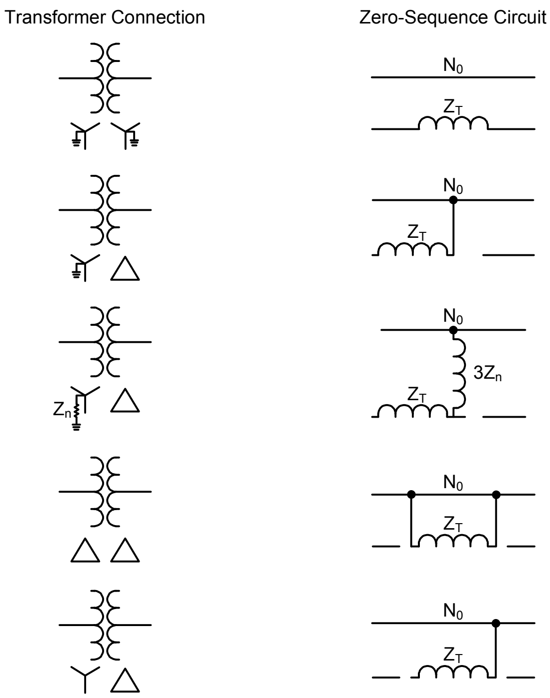

Fig. 1. Zero-sequence circuits for various transformer types

C. Connecting the Sequence Networks

Once the sequence networks for the system are defined, the way they are connected is dependent on the type of fault. Sequence network connections for common shunt fault types are shown in the remainder of this subsection. For complete derivations of these network connections as well as sequence network connections for series faults, see [2]. In the connections that follow, Z_F is defined as the fault impedance from each phase to the common point, and Z_G is defined as the impedance from the common point to ground. The Z_G term is only significant when Z_F differs per phase or if the line impedance to the fault point is different between phases. The typical assumptions are that Z_F is the same across all phases and the line impedances are equal, and therefore, the Z_G term is neglected.

For a three-phase fault, the positive-sequence network is used with the fault point connected back to the neutral bus, as shown in Fig. 2.

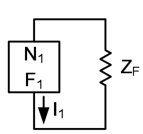

Fig. 2. Sequence network connections for a three-phase fault

For a single-phase-to-ground fault, the three networks are connected in series. Any fault impedance is multiplied by 3 and included in this connection, as shown in Fig. 3.

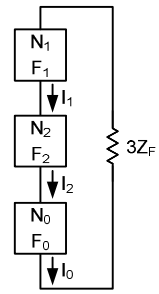

Fig. 3. Sequence network connections for a single-phase-to-ground fault

For a phase-to-phase fault, the positive- and negative-sequence networks are connected in parallel, as shown in Fig. 4.

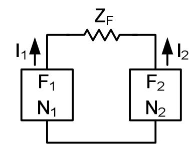

Fig. 4. Sequence network connections for a phase-to-phase fault

For a double-line-to-ground fault, all three networks are connected in parallel, as shown in Fig. 5.

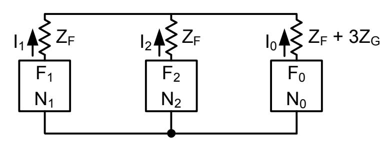

Fig. 5. Sequence network connections for a double-line-to-ground fault

D. The Per-Unit System

The per-unit system puts all the values of a power system on a common base so they can be easily compared across the entire system. To use the per-unit system, we normally begin by selecting a three-phase power base and a line-to-line voltage base. We can then calculate the current and impedance bases using the chosen power and voltage bases as shown:

$$I_{base} = \frac{S_{base}}{\sqrt{3} \cdot V_{base}} \tag{5}$$

$$Z_{base} = \frac{\left(V_{base}\right)^2}{S_{base}} \tag{6}$$

Any power system value can be converted to per unit by dividing the value by the base of the value, as shown:

$$\text{Quantity in per unit} = \frac{\text{Actual quantity}}{\text{Base value of quantity}} \tag{7}$$

Likewise, a per-unit value can be converted to an actual quantity at any time by multiplying the per-unit value by the base value of that quantity.

To convert impedances from one base to another, use the following equation:

$$Z_{pu}^{new} = Z_{pu}^{old} \bullet \frac{S_{base}^{new}}{S_{base}^{old}} \left(\frac{V_{base}^{old}}{V_{base}^{new}} \right)^2 \qquad (8)$$

For more information on the per-unit system, see [1].

E. Examples

The rest of this paper consists of theoretical and practical examples that can be used to practice and gain experience in symmetrical component and per-unit techniques. Each example consists of questions to guide the reader through the analysis as well as complete solutions. In the cases with real-world events, the event records from the relays are available for download with this white paper and the reader should use ACSELERATOR Analytic Assistant® SEL-5601 Software to view them (available for free download at http://www.selinc.com).

II. Example 1: Single-Phase Versus Three-Phase Fault Current

This example shows how to calculate fault currents for two different fault types at two different locations on a distribution system. Fig. 6 shows the radial system with two possible fault locations.

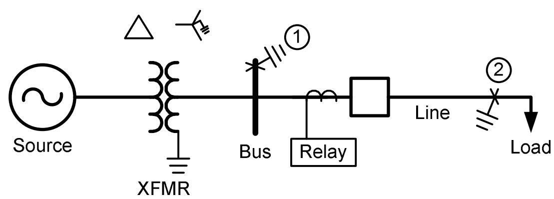

Fig. 6. Radial system with two fault locations

II-a On a radial distribution feeder, what type of fault do we expect to produce the largest fault current?

II-b Using symmetrical components, solve for the maximum fault current for a bolted three-phase fault at Location 1.

II-c Using symmetrical components, solve for the maximum fault current for a phase-to-ground fault at Location 1.

II-d Assume a core-type transformer with a zero-sequence impedance of 85 percent of the positive-sequence impedance. Solve for the fault current for a phase-to-ground fault at Location 1, and compare the results with that of a three-phase fault.

II-e Using symmetrical components, solve for the maximum fault current for a three-phase fault at Location 2.

II-f Using symmetrical components, solve for the maximum fault current for a phase-to-ground fault at Location 2. Is this greater than or less than the fault current for a three-phase fault?

III. Example 2: Per-Unit System and Fault Calculations

This example shows how to work in the per-unit system and calculate fault currents for faults at the high-voltage terminals of the step-up transformer shown in Fig. 7. The prefault voltage at the fault location is 70 kV_{LL}, and the generator and transformer are not connected to the rest of the power system. The source impedances shown are the subtransient reactances (X_d") of the generator [6].

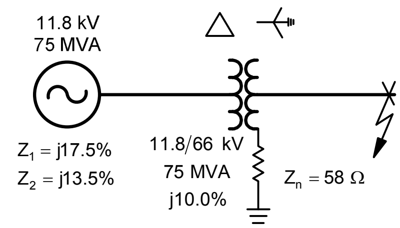

Fig. 7. One-line diagram for fault current calculations

III-a Select power and voltage bases for the per-unit system, and calculate current and impedance bases accordingly.

III-b Convert all impedances on the system as well as the prefault voltage to a common base.

III-c Draw the positive-, negative-, and zero-sequence networks for this system up to the fault point.

III-d What are the maximum short-circuit phase currents for a three-phase fault?

III-e What are the maximum short-circuit phase currents for a B-phase-to-C-phase fault?

III-f What are the maximum short-circuit phase currents for an A-phase-to-ground fault?

IV. Example 3: Fault Calculations for a Nonradial System

This example shows how to work in the per-unit system and calculate fault currents for a nonradial system, as shown in Fig. 8. The prefault voltage at the fault location is 1.05 per unit, and the load current is negligible. The source impedances shown are the subtransient reactances (X_d") of the generators [3].

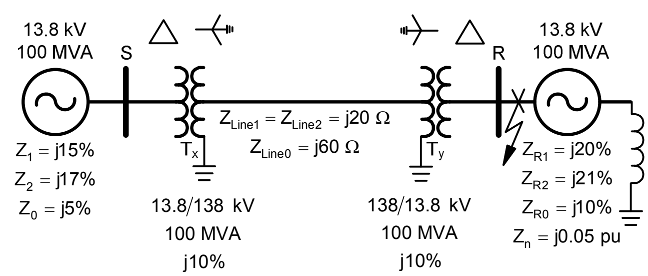

Fig. 8. One-line diagram of a nonradial system

IV-a Select power and voltage bases for the per-unit system, and calculate the current and impedance bases accordingly.

IV-b Convert all impedances on the system as well as the prefault voltage to a common base.

IV-c Draw the positive-, negative-, and zero-sequence networks for this system.

IV-d What are the maximum short-circuit phase currents for a three-phase fault?

IV-e What are the maximum short-circuit phase currents for a B-phase-to-C-phase fault?

IV-f What are the maximum short-circuit phase currents for a B-phase-to-C-phase-to-ground fault?

IV-g What are the maximum short-circuit phase currents for an A-phase-to-ground fault?

IV-h For an A-phase-to-ground fault, find the maximum positive-, negative-, and zero-sequence current contributions from Source S and Source R.

IV-i Find the phase voltages at the fault location during an A-phase-to-ground fault.

V. Example 4: Changing Bases

This example shows the importance of using the right base when computing symmetrical components. Typical textbook examples use an A-phase base, which always assumes an A-phase-to-ground, B-phase-to-C-phase, B-phase-to-C-phase-to-ground, or three-phase fault. For other fault types, the base will need to be changed accordingly in order to compute the correct symmetrical components.

This example shows a B-phase-to-ground fault that occurred on a transmission line. Open the event record titled **Example 4.cev**, and view the symmetrical components during the fault.

V-a Are the symmetrical component currents what we expect to see for a phase-to-ground fault?

V-b Derive the symmetrical components for an A-phase-to-ground fault.

V-c Derive the symmetrical components for a B-phase-to-ground fault.

V-d How do we obtain the correct symmetrical component values for a B-phase-to-ground fault?

V-e Why did the symmetrical components in ACSELERATOR Analytic Assistant not calculate correctly?

VI. Example 5: Phase Rotation

This example shows the importance of phase rotation when calculating sequence quantities. The event titled **Example 5.cev** is a simulated load condition on an SEL-351S Protection System. The trip equation in the relay is:

TR =51P1T + 51G1T + 67P1 + 50Q1 + OC

where 50Q1 is a negative-sequence instantaneous overcurrent element.

VI-a What is the pickup setting for 50Q1 in the relay? Based on the negative-sequence current seen in the event, should the relay have tripped?

VI-b Using the phase currents from the event, calculate the negative-sequence current I_2.

VI-c Is it normal to see this much negative-sequence current during unfaulted conditions?

VI-d What is the phase rotation of the system? Does this match the phase rotation setting in the relay?

VI-e Why is ACSELERATOR Analytic Assistant calculating high negative-sequence quantities?

VI-f Calculate the negative-sequence current by hand using ACB phase rotation.

VI-g Why did the relay not trip?

VII. Example 6: Fault Locator

This example shows how to use symmetrical components to determine a fault location using event reports from two ends of a transmission line. An internal single-line-to-ground fault was detected on a transmission line by the relays at both ends, as shown in Fig. 9. The event reports from each relay are provided in the event records titled **Example 6 - Side S.eve** and **Example 6 - Side R.eve**.

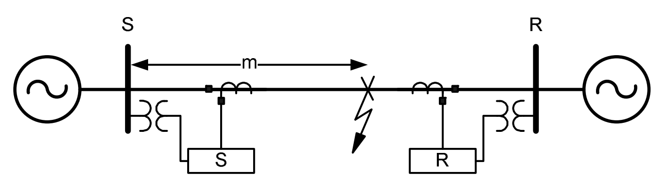

Fig. 9. Fault location on a two-source power system

VII-a Draw the sequence networks for this fault.

VII-b Using the sequence networks, write an equation to solve for the fault location *m*.

VII-c Use the event reports to obtain voltage and current values during the fault as well as the negative-sequence line impedance. Solve for *m*.

VIII. Example 7: Transformer Line-to-Ground Fault

This example shows how to derive the phase shift, symmetrical components, and fault currents across a delta-wye transformer. The event report titled **Example 7.cev** was generated after a current differential relay protecting a Dy1 transformer tripped, as shown in Fig. 10. Although the misoperation of the relay is not the focus of this exercise, it was caused by incorrect winding current compensation settings in the relay.

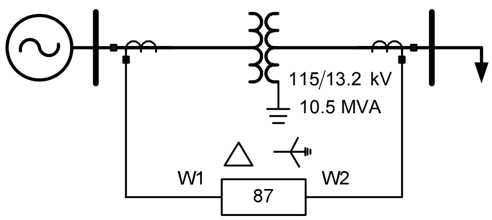

Fig. 10. Transformer current differential relay protecting a Dy1 transformer

VIII-a What type of fault is this? Assuming a radial system, is the fault internal or external to the zone of protection?

VIII-b Do we expect the prefault currents on the delta side to lead or lag the currents on the wye side?

VIII-c The transformer is connected to the system as shown in Fig. 11. Does this change the current lead/lag relationship we expect to see across the transformer? If so, how?

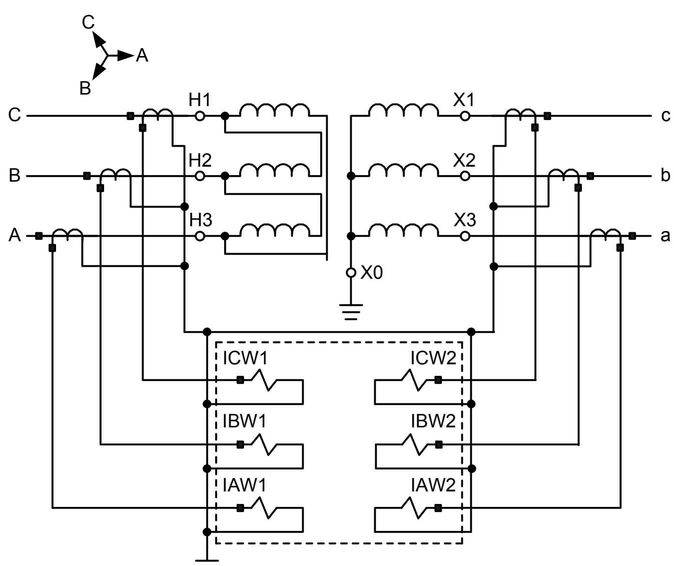

Fig. 11. Transformer phase-to-bushing connections

VIII-d Draw the phasors for the prefault currents we expect to see on the system as well as the currents coming into the relay. Do these match the prefault phasors in the event?

VIII-e Draw the phasors that we expect to see on the system as well as the phasors coming into the relay during the fault. Does this match what the event shows?

VIII-f Look at the symmetrical components in the event. Derive these phasors by drawing the sequence network of the fault.

VIII-g Using the sequence components, work backwards to derive the phase fault currents on the delta and wye sides of the transformer.

IX. Example 8: Transformer Phase-to-Phase Fault

This example shows how to derive the phase shift, symmetrical components, and fault currents across a delta-wye transformer. The event report titled **Example 8.txt** was generated after a current differential relay protecting a delta-wye transformer tripped, as shown in Fig. 12. The misoperation of the relay is not the focus of this exercise.

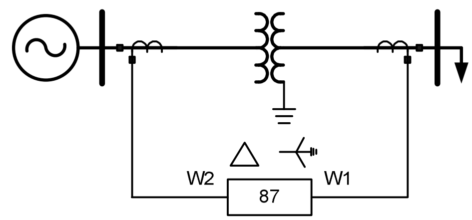

Fig. 12. Transformer current differential relay protecting a delta-wye transformer

IX-a What type of fault is this? Assuming a radial system, is the fault internal or external to the zone of protection?

IX-b The transformer is connected to the system as shown in Fig. 13. Do we expect the currents on the delta side to lead or lag the currents on the wye side?

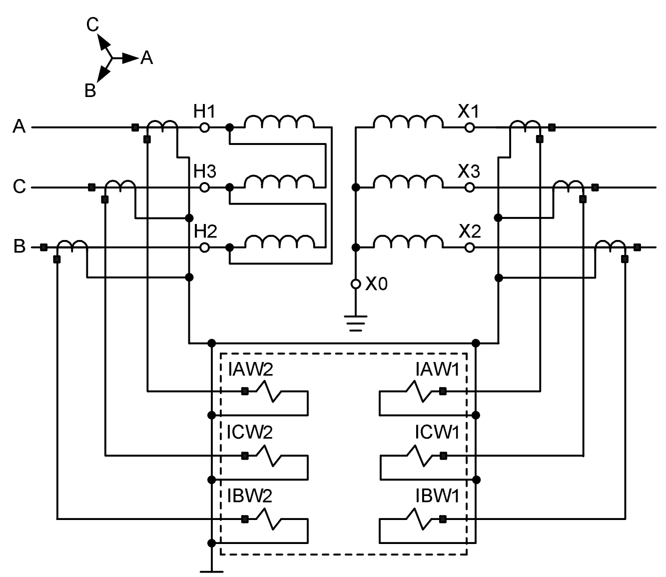

Fig. 13. Transformer phase-to-bushing connections

IX-c Draw the phasors for the prefault currents expected on the system as well as the phasors coming into the relay.

IX-d Draw the phasors expected on the system as well as coming into the relay during the fault. Does this match what the event shows?

IX-e Look at the sequence phasors in the event. Derive these phasors by drawing the sequence network of the fault.

IX-f Using the sequence components, work backwards to derive the phase fault currents on the delta and wye sides of the transformer.

X. References

[1] J. L. Blackburn and T. J. Domin, *Protective Relaying Principles and Applications*. CRC Press, Boca Raton, FL, 2007.

[2] P. M. Anderson, *Analysis of Faulted Power Systems*. Iowa State University Press, Ames, IA, 1973.

[3] J. D. Glover, M. S. Sarma, and T. J. Overbye, *Power System Analysis and Design (4th Edition)*. Thomson Learning, Toronto, ON, 2008.

[4] E. O. Schweitzer, III and Stanley E. Zocholl, "Introduction to Symmetrical Components," proceedings of the 58th Annual Georgia Tech Protective Relaying Conference, Atlanta, GA, April 2004.

[5] Westinghouse Electric Corporation, Relay-Instrument Division, *Applied Protective Relaying*. Newark, NJ, 1976.

[6] The Electricity Training Association, eds., *Power System Protection Volume 1: Principles and Components*. The Institution of Electrical Engineers, London, UK, 1995.

XI. Biographies

Ariana Amberg earned her BSEE, magna cum laude, from St. Mary's University in 2007. She graduated with a Masters of Engineering in Electrical Engineering from Texas A&M University in 2009, specializing in power systems. Ariana joined Schweitzer Engineering Laboratories, Inc. in 2009 as an associate field application engineer. She has been an IEEE member for 9 years.

Alex Rangel received a BSEE and an MSE from The University of Texas at Austin in 2009 and 2011, respectively. In January 2011, Alex joined Schweitzer Engineering Laboratories, Inc., where he works as an associate field application engineer. Alex is currently an IEEE member.

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